A Stochastic Approach for Modeling Machining Process Variables

by

Mohi uddin Ahmed

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

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DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
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In

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MOHI UDDIN AHMED

under the direction of his Thesis Committee, and approved by all the members, has been presented to and accepted by the Dean, College of Graduate Studies, in partial fullfilment of the requirements for the degree of

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Dr. Reda S. Al-Thiga
Dean, College of Graduate Studies

Date

Department Chairman

THESIS COMMITTEE

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S. Selim Feb 21, 84

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Anwar Khalil Sheikh

Member (Dr.A.K.Sheikh)

Dedicated to

the patience of

Loved Ones.

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ABSTRACT

In metal cutting industries and research organizations empirical models of machining process variables are developed for their use in machining optimization, adaptive control and computer aided manufacturing. The convential least square technique which has found a wide use in modeling estimates the mean values of the model parameters and therefore result in models which do not show the observed random nature of the various process responses. Since the variability in a process respons is expected to be reflected in the parameters of the model, it is essential to treat the model parameters as random variables associated with some probability distributions.

This thesis develops a statistical technique which can fit models whose parameters are random variables. To facilitate the implementation of the proposed technique a software package has been developed which identifies the distributions of the model parameters from the experimental data of the process response and the input variables.

Models which express the relationship between each of the four surface profile features namely, center-line-average, variance of heights, variance of slopes, and variance of curvatures, and the three fine turning machining variables,i.e

cutting speed, feed rate and depth of cut, are developed from experimental data using the proposed modeling technique. The Probability distributions of the surface profile features and the single, 2-factor, and 3-factor effects of the cutting conditons on the features are also investigated.

The newly developed models of the surface roughness features show that the proposed technique is capable of capturing the variablity of the machining responses which are not measureable by the conventional least square modeling technique.

خلاصة البحـــــــث

استنباط أسلوب احصائي لتطوير نماذج للمتغيرات العشوائية لعمليات قطع المعادن

فـــي صناعـــــة قطـــع المعـــادن وفـــي هيئـــات البحـــوث المتعلقـــــة بهـــــــا ، يتم تطويـــــر نمـــاذج تجريبيـــــة لمتغيـــرات عمليـــات القطـــع ليتـــم استخدامهـا فـــــي ايجـــاد ظـروف التشغيـل المثلــى وفي التحكم الانضباطي وفـــي طـــــــرق التصنيـع المدعومـــــة بالكمبيوتر . وبما أن الطريقة التقليدية الشائعة لتطوير النماذج بتقليل تربيع الخطأ تعطي القيـــــــم التقديرية لمتوسطات معالم النموذج ، فهي تقدم نماذجا لاتبين الطريقة العشوائية المعروفة عــــــــن متغيرات العملية المختلفة . وحيث أن التغيرية في متغير العملية يتوقع أن تنعكس في معالم النمــــوذج فانه من الضرورة معاملة معالم النموذج كمتغيرات عشوائية مرتبطه ببعض التوزيعات الاحتمالية .

طور هذا البحث أسلوبا احصائيا له المقدرة على استنباط نماذج تجريبية تكون معالمها متغيرات عشوائية ، ولتسهيل تطبيق الاسلوب المقترح تم ابتكار برنامج بالكمبيوتر يحدد توزيعات معالم النمـوذج من واقع البيانات التجريبية لاستجابة العملية ومتغيرات المعلومات الداخلة .

كما تم تطبيق هذا الاسلوب الاحصائي في تطوير نماذج تعبر عن العلاقة بين كل من أربعــــــة من خواص منحيات الجانبية وهي قيمة الخشونة المتوسطة ومتغير الارتفاع ومتغير الانحدار ومتغير التقـوس وبين ثلاثة متغيرات لعملية القطع هي سرعة القطع والتغذية وعمق القطع وذلك باستخدام بيانات تجريبيه من عملية الخراطة الناعمة . وقد تم بحث وايجاد التوزيعات الاحتمالية لكل من خواص منحنيـــــــــات الجانبيه والتأثيرات الفردية والثنائية والثلاثية لاحوال القطع بالخراطة الناعمة .

وتبين النماذج المطورة حديثا لظواهر الخشونه السطحيه بأن الاسلوب الاحصائي المقتـــــرح قادر على استنتاج متغيرات عملية القطع والتي لايمكن قياسها بالطرق التقليديه الشائعة لتطويـــــــــر النماذج بتقليل تربيع الخطأ .

1. INTRODUCTION

Mathematical models which express the relationship between a measured response and a number of input process variables, have been developed in various fields of science and engineering as well as in the social sciences. These models are needed for a number of applications such as for the selection of optimum process variables to keep a process output or response within desired limits. Empirical models are frequently helpful in operations that involve interpolation , extrapolation, linearization, differentiation, and integration.

Like other engineering desciplines, metal cutting or machining industries need to develop empirical models which express the relationship between their output machining variables (e.g., tool life, surface-roughness, cutting force, etc.) and the process cutting conditions, such as cutting speed, feed rate, tool-geometry, depth of cut, etc. This is due in part to the continuing demand for their use in machining optimization and in the field of computer aided and adaptive control machining. In the former application, the models are used to determine the optimal operating conditions whereas in the latter, the machining process is controlled by adjusting the cutting conditions to maintain a certain output machining variable within desired limits.

The empirical models under consideration in this study relate the output response 'R' of any machining process to its cutting conditions $Z_1, Z_2, \ldots, Z_n$ by the following expression;

$$R = KZ_1^{a_1} Z_2^{a_2} \ldots Z_n^{a_n} \tag{1.1}$$

where $K, a_1, a_2, \ldots a_n$ are the parameters of the model, which are estimated by a suitable parameter estimation technique. Many technical papers [1-20] describe the development of the above model, by estimating its parameters using the conventional least square or regression analysis technique.

The conventional parameter estimation techniques usually estimate the mean values of the model parameters and their respective confidence limits [21]. In the application of the models obtained using the conventional techniques it has been observed that experimental values of the machining response 'R' shows a considerable scatter in comparison with the corresponding estimated values obtained from the models [22-26] . This scatter is explained by the inherent variations in the machining process due to such factors as tool wear, built-up edge, chatter, tool and workpiece material properties, etc. These factors cause the output machining variables to vary in a random manner. Since the random nature of the machining process is expected to be reflected in the corresponding model parameters[27], it is concluded that the model parameters $K, a_1, a_2, \ldots a_n$ are random variables which are to be characterized by their respective statistical distributions and the related distribution parameters. Therefore, the objectives of this thesis are twofold:

a) The development of a statistical modeling technique capable of identifying the distributions of the random parameters of the model and obtaining the values of the identified distribution parameters. The technique uses field or experimental data of the response and the input process variables to compute a set of moments for each parameter of the model. A criterion to identify the distributions of the model parameters which is based on comparing the moment ratios of the computed moments and of the theoretical moment ratios of the distributions has been developed.

b) The development of fine turning surface roughness models using the proposed statistical modeling technique and the fine turned surface roughness experimental data. Four models are developed to express the functional relationship between the random surface profile characteristic parameters, namely; center-line-average R_a, variance of heights m_0, variance of slopes m_2 and variance of curvature m_4, and the three input machining variables (cutting speed, feed rate and depth of cut). The application of the technique to the modeling of surface profile characteristic parameters has been dictated by the fact that, these responses are random and their model parameters are therefore expected to be random variables characterized by specific distribution and respective distribution parameters.

Chapter 2 of this thesis gives an overview of the literature survey. The chapter includes a brief information about the state of the art of the empirical models developed by various metal cutting researchers.

Chapter 3 outlines the development of the statistical modeling technique which estimates a set of moments for each of the model random parameter from experimental data. The set of moments are then used for identifying the distribution of the respective model random parameters. The chapter also includes the verification of the developed technique through Monte-Carlo simulation.

The experimental work performed to generate surface roughness data for a fine turning operation is described in chapter 4. The experimental work includes turning of EN08 steel bars at different sets of cutting conditions, generation and recording of analog surface profile data, digitization and acquisition of the profile data, and the investigation of optimum sampling interval for digitizing the analog profile signals.

Chapter 5 uses the experimental data generated in Chapter 4, to develop the surface-roughness models by applying the statistical modeling technique. The distributions of each model random parameter are identified. The conventional least square parameter estimation technique is compared with the statistical modeling technique. The comparison has shown that the technique offers a method to identify the distributions of the model parameters and, therefore, caters for the random nature of the responses. The effects of machining conditions and their interactions are also studied and both the main effects and the higher order interactions are found to be significant.

Chapter 6 outlines the conclusions and recommendations arising from the present work.

2. LITERATURE REVIEW

It was stated in Cahpter 1 that the objective of this thesis is (a) to develop a technique which can model the machining process variables and (b) to use the technique to develop fine turning surface roughness models. This chapter presents an overview of the various machining process models developed and used in the metal cutting industry.

An extensive review of the literature shows that researchers [1-22] in the metal cutting area have developed both physical and empirical models for various machining processes. Physical models are derived from physical laws and therefore have physical meanings. However, these models are characterized by the limitation that they are difficult to use in complex problems without oversimplication. Empirical models characterize the functional relationship between the machining process response and its cutting conditions directly through a set of experiments. The works reported by various researchers have used statistically planned experiments to develop statistically adequate models of the output machining process response in terms of the process cutting conditions.

The following sections will review a number of empirical models developed for a number of machining process variables.

TOOL LIFE MODELS

The first empirical model on tool life [1] was introduced by F.W. Taylor in the year 1907. The relationship between the cutting speed V, and tool life T, was given by the equation

$$VT^n = C$$

where n and C are constant values for a definite set of conditions. In the mid-1960's the techniques, including factorial, fractional-factorial and response surface methodology, were applied to tool life experiments by Wu [2]. Later other forms of the empirical tool life models were developed from experimental data [3-4]. These models include

1- Taylor model $\ell nT = b_0 + b_1(\ell nV)$

2- Extended-Taylor model $\ell nT = b_0 + b_1(\ell nV) + b_2(\ell nF) + b_3(\ell nD)$

3- Second Order model $\ell nT = b_0 + b_1(\ell nV) + b_2(\ell nF) + b_3(\ell nD) + b_{11}(\ell nV) + b_{22}(\ell nF) + b_{33}(\ell nD) + b_{12}(\ell nV)(\ell nF) + b_{13}(\ell nV)(\ell nD) + b_{23}(\ell nF)(\ell nD)$

where V, F and D are the cutting speed, feed rate and depth of cut respectively; and bi's are the model parameters whose mean values are estimated using the least square technique. Since the estimated mean values of the model parameters result in a single value of the tool life at a specific set of cutting conditions the above tool life models do not show the observed scatter in the tool life.

SURFACE ROUGHNESS MODELS

Out of the numerous machining responses, the surface roughness has found a wide consideration in the field of metal cutting research.

The effect of cutting conditions (cutting speed, feed rate, depth of cut and tool nose radius) on surface roughness profile parameters in turning operation was investigated by Hossgaw et al [5] by using the response surface methodology. An empirical model for peak-to-valley height of the surface profile, R_t, in terms of the cutting conditions was developed and the least square technique was used for estimating the parameters of the model, from experimental data.

Nassipour and Wu [6] developed first and higher order models for different surface profile characteristic parameters in terms of the cutting speed, feed rate and tool nose radius. A 3^3 factorial design experiment was conducted to estimate the model parameters by using regression analysis.

Sundaram and Lambert [7] developed the first and higher order log-transformed models for predicting the surface finish of AISI 4140 steel in the file turning operation using TiC coated tungsten carbide throw away tools. A novel experimental design called the rotatable design was used for generating the experimental data. The process variables included in the model were cutting speed, feed rate, depth of cut and time of cut of the tool. Multiple regression analysis was used for estimating the parameters of the models. Since the mean values of the model parameters were estimated, the true picture of the machining process which causes the random variation in the surface finish due to such factors as chatter, vibration, built-up edge, etc., would not be reflected by these models.

CUTTING FORCE MODELS

Another important machining variable which has been modeled empirically is the cutting force. Several empirical equations had been presented by 1950's and some of these are still in use. Since cutting force equations were based on relatively old cutting data, several problems may arise when they are applied to the present cutting conditions. Inspite of the large number of papers published as a result of the theoretical study of metal cutting, the consideration of the random nature of cutting force during machining is not common.

That is why, metal cutting theory cannot accurately predict the cutting force for practical cutting operations.

Moltrecht [14] has derived analytical equations for estimating the three components of the cutting force in terms of the cutting conditions and the horse power for different machining operations. Nakayama and Arai [15] have used a very simple method for the prediction of the three components of cutting forces. The method requires six cutting variables (speed, feed, depth of cut, rate angle, side cutting edge angle and corner radius of cutting tool) to be used in the empirical and analytical equations. These equations are then used for estimating the cutting forces.

Lambert et al [16] have developed a cutting force equation for single-point orthogonal cutting which incorporates cutting speed, feed, depth of cut and side rake angle. Regression analysis technique was used for estimating the model parameters from cutting force experimental data. It was concluded that 93.3 percent of the predicted values of the cutting force were within ± 25 percent of the actual experimental force values, 81.7 percent of the predicted forces were within ± 20 percent of the actual values; 66.7 percent of the predicted values were within ± 15 percent of the actual values and 56.7 percent of the predicted values were within ± 10 percent of the actual cutting force values. This large variation of predicted values from the actual experimental values show the random nature of cutting forces during the machining process. The developed model results in only one value of the cutting force at a specific set of machining conditions, inspite of the observed large variation.

An empirical model of the cutting force acting on a carbide tool in a turning operation was constructed by Lambert et al [17]. Cutting speed, feed rate and depth of cut were used as the input machining variables. The developed model was then utilized to select the levels of the machining variables such that the rate of metal removal could be considerably increased without increasing the cutting force. Since the mean values of the model parameters were estimated using the least square method only one value of the cutting force is given by the model at a given set of cutting conditions.

TORQUE, THRUST AND POWER MODELS

The responses of torque, thrust, power, etc., have found a wide consideration in the machining processes. Empirical models were constructed by Milner and Raafat [18] giving the relationship between these responses and the input variables such as material hardness, cutting speed, feed rate and the number of teeth on the gear, for a gear hobbing process. Multiple regression analysis was used for estimating the model parameters. The conclusion is based on the effect of cutting speed and feed rate on both the torque on the hobbing machine shaft and the average power consumed during the process. Generally it was observed that the relative vibrations between the hobbing tool and workpiece caused the variation in the torque as well as in the power, but this factor was not considered during the development of models.

Lambert [19] developed the first and second order empirical models for a drilling process to relate the thrust, torque and burr height to the cutting speed, feed, point angle and relief angle as the machining conditions. A 2^4 factorial design experiment was conducted by ordinary linear least square method. The regression models developed in the study provide good predictors of drill performance, but they did not incorporate the random nature of the drilling process.

WEAR MODELS

The inevitable wear of cutting tools has been the subject of many investigations during the past century with numerous articles having appeared in the technical literature on this subject. DeVries [20] proposed a single empirical formula to describe the wear of carbide cutting tools with input variables such as temperature and cutting speed. Both flank and crater wear was characterized by the same model form, with only the parameters in the equation differing. The values of the parameters in the model were based on the experimental data collected on the cutting of medium carbon AISI 1048 steel with a single carbide grade (KG) and the application of least square parameter estimation method.

From the overall review of the above literature, the problem which can be emphasized at the outset of this investigation is summarized as follows:

1. Most of the above models have been developed on the basis of laboratory tests primarily designed to use the conventional least square model building technique. Since only the mean values of the parameters of the models, estimated using this technique [28] , the output responses estimated from the models with these parameters do not reveal the random variation in the machining processes.

2. Since the reported random nature of the machining process is expected to be reflected in the corresponding model parameters, it is therefore necessary to develop a modeling technique which can estimate these random parameters and identify their respective statistical distributions.

3. DEVELOPMENT OF THE STATISTICAL MODELING TECHNIQUE

Engineers and scientists experience a continuous need for developing empirical models which express the functional relationship between a response and one or more independent process variables. For example in metal cutting industries, models that relate the response (e.g., surface finish) to the input process variables (i.e., cutting conditions) have been developed by researchers [5-13]. The conventional methods used to estimate the model parameters such as least square estimation technique, are characterized by the limitation that only the mean values of the model parameters are estimated.

Since the response for which empirical models are to be established are often found to be random variables, the development of models which take care of the random variation of the responses is highly desirable. Of course, the random nature of the response indicates that the model parameters have to be random variables.

The objective of this chapter is the development of a statistical modeling technique which can fit models whose parameters are treated as random variables. The proposed technique has to identify the statistical distributions of the model random parameters and obtain their respective distribution parameters. The technique requires experimental or field data to estimate a set of moments for each random parameter of the model. The respective distribution of the parameters are identified from the computed set of moments using a proposed criterion of moment ratios.

A review of the theoretical basis of the proposed modeling technique is presented in Section 3.1 whereas the development of the technique is covered in Section 3.2. The criterion for identifying statistically adequate distributions for the model random parameters is established in Section 3.3. A description of the computer program used in the development of the model from experimental data is presented in Section 3.4. Section 3.5 presents a demonstration and verification of the developed technique, using simulated data. The results obtained during the technique verification are discussed in Section 3.6 and finally relevant conclusions are drawn in Section 3.7.

3.1 THE THEORETICAL BASIS OF THE PROPOSED STATISTICAL MODELING TECHNIQUE

This section overviews the mathematical background of the proposed statistical modeling technique which is based on specifying the distributions of the model random parameters when their respective moments are known. The moment generating function (mgf) which has found a wide application in probability theory is used to develop the relationship between the moments of a response and the moments of the model random parameters. An overview of the moment generating function and some of its important properties are covered below. Special emphasis is given to the property which is used to relate the moments of the response to the moments of the model parameters.

3.1.1 The Moment Generating Function

Suppose X is a random variable with a given distribution, $F_X(x) = \int_0^x f_X(x)\,dx$, and that, for some real number $h > 0$, $E(e^{tX})$ exists for every value of t in the interval $(-h, h)$. Then the function M defined by

$$M_X(t) = E(e^{tx})$$

is called the moment generating function (mgf) of X or of the distribution of X. Thus

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f_X(x)\,dx \tag{3.1}$$

if X is a continuous random variable, and

$$M_X(t) = \sum_{x_i} e^{tx_i} P(X = x_i) \tag{3.2}$$

if X is a discrete random variable.

The following derivation [29] will show, how the mgf can be used to obtain the moments of the distribution of X. Since the Maclaurin's series expansion of e^x is

$$e^x = 1 + x + x^2/2! + x^3/3! + \dots$$

which converges for all x, we can write

$$e^{tX} = \sum_{r=0}^{\infty} \frac{(tx)^r}{r!} \tag{3.3}$$

If $M_X(t)$ exists for $t \varepsilon (-h, h)$ for some positive number h, we can write

$$M_X(t) = E(e^{tX}) = E\left(\sum_{r=0}^{\infty} \frac{(tX)^r}{r!}\right) \tag{3.4}$$

and interchanging the order of expectation and summation results in:

$$M_X(t) = \sum_{r=0}^{\infty} E(X^r)\cdot\frac{t^r}{r!} \tag{3.5}$$

Thus the coefficient of $t^r/r!$ in the power series expansion of $M_X(t)$ is precisely $E(X^r)$, which is the rth moment of X about the origin, defined by μ'_r.

On the other hand, the Maclaurin's series for $M_X(t)$ is:

$$M_X(t) = \sum_{r=0}^{\infty} M_X^{(r)}(0)\cdot\frac{t^r}{r!} \tag{3.6}$$

using the standard notation $M_X^{(r)}(0)$ in place of $\frac{d^r}{dt^r} M_X(t)\Big|_{t=0}$.

Comparing the coefficient of t^r in Equations 3.5 and 3.6, it follows that

$$E(X^r) = M_X^{(r)}(0), \qquad r = 1, 2, \ldots .$$

The preceding derivations show that when the mgf of a random variable X is available, it can be used in two ways to find the moments of the distribution:

(i) if the mgf can be expanded as a power series of t, then the coefficient of $t^r/r!$ gives $E(X^r)$;

(ii) if the mgf can be differentiated repeatedly, then the rth order derivative evaluated at $t = 0$ is $E(X^r)$.

As an example the mgf and the moments of a standard normal distribution $N(0, 1)$ with probability density function $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ can be derived as follows:

Using the definition of the mgf

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f_X(x)\, dx = e^{t^2/2} \qquad (3.7)$$

is the mgf of standard normal distribution. The moments are obtained from the power series expansion of Equation (3.7):

$$M_X(t) = e^{t^2/2} = \sum_{r=0}^{\infty} \left(\frac{t^2}{2}\right)^r \cdot \frac{1}{r!}$$

$$= \sum_{r=0}^{\infty} \frac{(2r)!}{2^r r!} \cdot \frac{t^{2r}}{(2r)!}$$

The coefficient of $t^{2r}/(2r)!$ is $\dfrac{2r!}{2^r r!}$ therefore

$$E(X^{2r}) = \frac{2r!}{2^r r!} \qquad r = 0, 1, 2, \ldots \tag{3.8}$$

and for odd order moments the coefficient of t^{2r+1} is zero for every non-negative integer r; therefore,

$$E(X^{2r+1}) = 0 \qquad r = 0, 1, 2, \ldots . \tag{3.9}$$

Alternatively, the moments can be obtained by differentiating the mgf in the following way:

$$\frac{d}{dt} M_X^{(t)} = t\, e^{t^2/2}$$

$$\frac{d}{dt} M_X(t)\Big|_{t=0} = E(X) = 0$$

and

$$\frac{d^2}{dt^2} M_X(t) = (1 + t^2)\, e^{t^2/2}$$

$$\frac{d^2}{dt^2} M_X(t)\Big|_{t=0} = E(X^2) = 1$$

From Equation 3.8

$$E(X^{2r}) = \frac{2r!}{2^r r!}, \quad E(X^2) = 1$$

and

$$E(X^r) = \frac{d^r}{dt^r} M_X(t) \Big|_{t=0}, \tag{3.10}$$

is the rth moment expression for the distribution. It is thus possible to obtain the expression for the rth moment of any other distribution using either the power series or the repeated differentiation approaches. A list of the rth moments were compiled from [30] for seven common continuous distributions and listed in Table 3.1.

In general, the expected value $E(X^r)$ is referred to as the rth moment of random variable X about the origin and is denoted by μ'_{rX}. Letting $r = 1$ yields the first moment of X about the origin, that is, $\mu'_1 = \mu$ where μ is the mean value of X. For many statistical analysis, the moments of X about the mean are very useful. These moments are also called the central moments and are denoted by μ_{rx} ($\mu_{rx} = E(x - \mu)^r$). The recurrence relations between the moments about the origin and moments about the mean is given by

$$\text{a)} \quad \mu'_r = \sum_{i=0}^{r} \binom{r}{i} \mu_{r-i} (\mu'_1)^i$$

$$\text{b)} \quad \mu_r = \sum_{i=0}^{r} \binom{r}{i} \mu'_{r-i} (-\mu'_1)^i \tag{3.11}$$

TABLE 3.1 The rth Moment Expression for Some Continuous Distributions

No.	Distribution	Probability Density Function $f_X(x)$	Distribution Parameters	rth Moment $r = 1,2,3,\ldots$
1.	Normal	$\frac{1}{\sigma\sqrt{2\pi}} \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}$	μ = location σ = scale	$\mu_{2r+1} = 0 \quad r = 0,1,2,3,\ldots.$ $\mu_{2r} = \frac{2r!}{(2)^r(r!)}(\sigma)^{2r}$
2.	Lognormal	$\frac{1}{\sigma x\sqrt{2\pi}} \exp\{-\frac{(\ln x-\theta)^2}{2\sigma^2}\}$	θ = scale σ = shape	$\mu'_r = \exp[r\theta + 0.5r^2\sigma^2]$
3.	Uniform	$\frac{1}{b-a} \quad a \leq x \leq b$	a = location b-a = scale	$\mu_{2r-1} = 0 \quad r = 0,1,2,3,\ldots.$ $\mu_{2r} = (b-a)^{2r}/2^{2r}(2r+1)$
4.	Exponential	$\lambda \exp(-\lambda x)$	$1/\lambda$ = scale	$\mu'_r = r!/\lambda^r$
5.	Chi-Square	$\frac{\exp(-x/2)(x)^{(\lambda-1)}}{(2)^\lambda\Gamma(\lambda)}$	2λ = mean	$\mu'_r = 2^r \prod_{i=0}^{r-1}(i+\lambda)$
6.	Gamma	$\theta^\alpha x^{(\alpha-1)}\exp(-\theta x)/\Gamma(\alpha)$	α = shape θ = scale	$\mu'_r = \frac{1}{r}\prod_{i=0}^{r-1}(\alpha+i)$
7.	Beta	$((1-x)^{\theta-1}x^{\alpha-1})/\beta(\alpha,\theta)$	α = shape θ = scale	$\mu'_r = \prod_{i=0}^{r-1}\frac{\alpha+i}{\alpha+\theta+i}$

μ'_r = rth moment about the origin $\qquad \mu_r$ = rth moment about the mean where

$$r = 1, 2, 3, \ldots$$

$$\mu_0 = \mu_0' = 1$$

$$\mu_1 = 0$$

3.1.2 Properties of the Moment Generating Function:

The mgf has some interesting properties which are very useful in probability theory [29]. One of the properties which is going to be used in the development of the statistical modeling technique is the reproductive property. Suppose that a response Y is expressed as a function of a set of independent process variables $X_1, X_2, \ldots, X_n$ and a set of random parameters $a_0, a_1, a_2, \ldots, a_n$ using:

$$Y = a_0 + a_1 X_1 + \ldots a_n X_n \tag{3.12}$$

then using the mgf definition for Y

$$\begin{aligned} M_Y(t) &= E(e^{tY}) \\ &= E\left[e^{(a_0+a_1X_1+a_2X_2 \ldots a_nX_n)t}\right] \\ &= E(e^{a_0t})\cdot E(e^{a_1X_1t})\cdot E(e^{a_2X_2t}) \ldots E(e^{a_nX_nt}) \end{aligned} \tag{3.13}$$

$$M_Y(t) = M_{a_0}(t)\cdot M_{a_1X_1}(t)\cdot M_{a_2X_2}(t) \ \ldots \ M_{a_nX_n}(t) \, .$$

Since $X_1, X_2 \ldots X_n$ are constants and $a_0, a_1 \ldots a_n$ are random variables, then

$$M_Y(t) = M_{a_0}(t) \cdot M_{a_1}(X_1 t) \cdot M_{a_2}(X_2 t) \ldots M_{a_n}(X_n t). \tag{3.14}$$

Equation 3.14 indicates that for a linear model represented by Equation 3.12 the moment generating function of the response Y is equal to the product of the moment generating functions of the model random parameters.

3.2 THE PROPOSED STATISTICAL MODELING TECHNIQUE:

After describing the essential mathematical background in Section 3.1, the proposed modeling technique will be discussed in this section. The model proposed for this technique is of the following form:

$$R = KZ_1^{a_1} Z_2^{a_2} \ldots Z_n^{a_n} \tag{3.15}$$

where

R = the response of the process (e.g., vibration, force, surface roughness parameter, etc).

$Z_1, Z_2, \ldots, Z_n$ = the process independent variables (e.g., cutting speed, feed rate, depth of cut, etc.)

and $K, a_1, a_2, \ldots, a_n$ = the parameters of the model which are random variables.

The proposed technique is to be used to identify the distributions of the model parameters and estimate the respective distribution parameters. To apply the technique, the model is linearized by taking the natural logorithm of both sides of Equation 3.15:

$$\ell n\ R = \ell n\ k + \sum_{i=1}^{n} a_i\ \ell n\ Z_i$$

or

$$Y = a_0 + \sum_{i=1}^{n} a_i X_i \tag{3.16}$$

where

$$Y = \ell n\ R$$

$$a_0 = \ell n\ K$$

$$X_i = \ell n\ Z_i, \quad i = 1, 2, \ldots, n.$$

It has been stated in Chapter 1 that most processes result in a significant scatter in the values of the response Y measured at a specific set of the independent variables $X_1, X_2, \ldots, X_n$. The scatter in Y has been attributed to the inherent variations in the process (Chapter 1). Since the random nature of Y is expected to be reflected in the corresponding model parameters, the model parameters a_0, a_1, a_2 ... a_n are random variables whose properties and statistical distributions have to be specified. By applying the reproductive property of the mgf of Section 3.1.2, it is possible to get the mgf of Equation 3.16 as follows:

$$M_Y(t) = M_{a_0}(t) \cdot M_{a_1 X_1}(t) \cdot M_{a_2 X_2}(t) \ \ldots \ M_{a_n X_n}(t) \tag{3.17}$$

and

$$M_Y(t) = M_{a_0}(t) \cdot M_{a_1}(X_1 t) \cdot M_{a_2}(X_2 t) \ \ldots \ M_{a_n}(X_n t) \tag{3.18}$$

The expressions relating the moments of Y and moments of a_0, a_1 ... a_n will be derived below by applying the repeated differentiation method on Equation 3.18.

The relationship between the first moment of the response Y and the first moments of a_0, a_1 ... a_n is obtained by taking the first derivative of Equation 3.18 with respect to t and then setting $t = 0$:

$$M_Y'(t) = \frac{d}{dt} M_Y(t)$$

$$M_Y'(t) = \{M_{a_0}'(t) \cdot M_{a_1}(X_1 t) \cdot M_{a_2}(X_2 t) \ \ldots \ M_{a_n}(X_n t)\}$$

$$+ \{M_{a_0}(t) \cdot M'_{a_1}(X_1 t) X_1 \cdot M_{a_2}(X_2 t) \ \ldots \ M_{a_n}(X_n t)\}$$

$$+ \{ \qquad \ldots \qquad \ldots \qquad \}$$

$$+ \{ \qquad \ldots \qquad \ldots \qquad \}$$

$$+ M_{a_0}(t) \cdot M_{a_1}(X_1 t) \cdot M_{a_2}(X_2 t) \ \ldots \ M'_{a_n}(X_n t) X \ )$$

Setting $t = 0$ and using

$$M_X(0) = 1$$

$$M'_X(0) = \mu'_{1X}$$

we get:

$$\mu'_{1Y} = \mu'_{1a_0} + \mu'_{1a_1} X_1 + \mu'_{1a_2} X_2 + \ldots \mu'_{1a_n} X_n \qquad (3.19)$$

where

$\mu'_{1Y}, \mu'_{1a_0} \cdots \mu'_{1a_n}$ are the first moments about the origin of Y, a_0, a_1 .. a_n respectively.

Equation 3.19 is a linear equation which expresses the first moment of Y as a function of the first moments of the random parameters a_0, a_1 ... a_n and the process variables X_1, X_2 ... X_n. At each set of cutting conditions of X_1, X_2, ..., X_n the moments about the origin of Y ($\mu'_{1y}, \mu'_{2y}, \ldots \mu'_{ry}$) are computed from NR measurements of the response Y. To apply Equation 3.19, a number of NE tests have to be conducted so that their first moments of Y and the respective cutting conditions can be used to estimate μ'_{1a_0}, μ'_{1a_1}, ..., μ'_{1a_n}. Table 3.2 summarizes the data format required for estimating the random parameter moments from the response moments.

Now by taking the second derivative of Equation 3.18, with respect to t and setting $t = 0$, the second moment of Y about origin (μ'_{2Y}) in terms of the process variables X_1, X_2 ... X_n and the second moments about origin of a_0, a_1 ... a_n, can be obtained as:

$$\begin{aligned}
\mu'_{2Y} = {} & \mu'_{2a_0} + \mu'_{2a_1} X_1^2 + \mu'_{2a_2} X_2^2 \ldots \mu'_{2a_n} X_n^2 \\
& + 2\mu'_{1a_0}(\mu'_{1a_1} X_1 + \mu'_{1a_2} X_2 + \ldots \mu'_{1a_n} X_n) \\
& + 2\mu'_{1a_1} X_1 (\mu'_{1a_2} X_2 + \mu'_{1a_3} X_3 + \ldots \mu'_{1a_n} X_n) \\
& + 2\mu'_{1a_2} X_2 (\mu'_{1a_3} X_3 + \mu'_{1a} X_4 + \ldots \mu'_{1a_n} X_n) \\
& + \qquad \ldots \qquad \ldots \\
& + \qquad \ldots \qquad \ldots \\
& + 2\mu'_{1a_{n-1}} X_{n-1} (\mu'_{1a_n} X_n)
\end{aligned} \tag{3.20}$$

where

TABLE 3.2 Data Format for Estimating the Moments of $a_0, a_1, a_2, \ldots, a_n$ From the Moments of the Response Y

Expt. No.	Values of Process Variables [x]	Responses y's Measured [y]	rth Moments of Response y's estimated. [UY]
1	$X_{1,1}\ X_{2,1} \cdots X_{n,1}$	$y_{1,1}, y_{2,1}, y_{3,1} \cdots y_{NR,1}$	$\hat{\mu}'_{1y_1}, \hat{\mu}'_{2y_1}, \cdots \hat{\mu}'_{ry_1}$
2	$X_{1,2}\ X_{2,2} \cdots X_{n,2}$	$y_{1,2}, y_{2,2}, y_{3,2} \cdots y_{NR,2}$	$\hat{\mu}'_{1y_2}, \hat{\mu}'_{2y_2}, \cdots \hat{\mu}'_{ry_2}$
3	$X_{1,3}\ X_{2,3}\ \ X_{n,3}$	$y_{1,3}, y_{2,3}, y_{3,3} \cdots y_{NR,3}$	$\hat{\mu}'_{1y_3}, \hat{\mu}'_{2y_3}, \cdots \hat{\mu}'_{ry_3}$
.			
.			
.			
.			
NE	$X_{1,NE}\ X_{2,NE}\ \ X_{n,NE}$	$y_{1,NE}, y_{2,NE}, y_{3,NE} \cdots y_{NR,NE}$	$\hat{\mu}'_{1y_{NE}}, \hat{\mu}'_{2y_{NE}}, \cdots \hat{\mu}'_{ry_{NE}}$

$\mu'_{2Y}, \mu'_{2a_0}, \mu'_{2a_1} \dots \mu'_{2a_n}$ are the second moments of Y, a_0, $a_1 \dots a_n$ about the origin.

Letting

$$G_2 = 2\mu'_{1a_0}(\mu'_{1a_1}X_1 + \mu'_{1a_2}X_2 \dots \mu'_{1a_n}X_n)$$
$$+ 2\mu'_{1a_1}X_1(\mu'_{1a_2}X_2 + \mu'_{1a_3}X_3 + \dots \mu'_{1a_n}X_n)$$
$$+ 2\mu'_{1a_2}X_2(\mu'_{1a_3}X_3 + \mu'_{1a_4}X_4 + \dots \mu'_{1a_n}X_n)$$
$$+ \quad \dots \quad \dots$$
$$+ \quad \dots \quad \dots$$
$$+ 2\mu'_{1_{n-1}}X_{n-1}(\mu'_{1a_n}X_n) \tag{3.21}$$

Equation (3.20) becomes:

$$\mu'_{2Y}-G_2 = \mu'_{2a_0} + \mu'_{2a_1}X_1^2 + \mu'_{2a_2}X_2^2 + \dots \mu'_{2a_n}X_n \tag{3.22}$$

Equation 3.22 is a linear equation which relates the dependent variable $(\mu'_{2Y}-G_2)$ to the independent variables X_1^2, $X_2^2 \dots X_n^2$. It is possible again to estimate the second moments μ'_{2a_0}, $\mu'_{2a_1} \dots \mu'_{2_{a_n}}$ by using the ordinary least square estimation method. The second moment of Y, μ'_{2Y}, has to be estimated at each set of X_1, $X_2 \dots X_n$ as shown in Table 3.2, and G_2 has to be calculated using Equation (3.21).

Similarly, by taking the third, fourth, fifth ... rth derivative of Equation 3.18, with respect to t and setting $t = 0$, the third, fourth, fifth ... rth moments about origin of a_0, a_1 ... a_n can be evaluated. A closer examination of the above derivation shows that the expression of the linear relationship between the moments of response Y and the moments of the random parameters a_0, a_1 ... a_n and the process variables X_1, X_2 ... X_n, can be expressed in a generalized series form:

$$\mu'_{rY} = \{ \sum_{i=0}^{r} \sum_{j=0}^{r} \cdots \sum_{m}^{r} (\frac{r!}{i!\ j!\ \ldots\ m!}) \} .$$

$$\{ (\mu'_{ia_0}) \ (\mu'_{ja_1} X_1^j) \ \ldots \ (\mu_{ma_n} X_n^m) \} \tag{3.23}$$

where

$r = 1, 2, 3, \ldots$

$m = r - i - j \geq 0.$

On expanding the series of Equation 3.23, it is possible to obtain the following relationship

$$\mu'_{rY} - G_r = \mu'_{ra_0} + \mu'_{ra_1} X_1^r + \mu'_{ra_2} X_2^r + \ldots \mu_{ra_n} X_n^r \tag{3.24}$$

where; μ'_{rY}, μ'_{ra_0}, μ'_{ra_1} ... μ'_{ra_n} are the rth moments of Y, a_0, a_1 ... a_n respectively and $G_r = f(\mu_{1a_i}\ \mu_{2a_i}\ \ldots\ \mu_{r-1}a_i,\ X_1,\ X_2\ \ldots\ X_n)$.

In Section 3.4, a computer algorithm is developed to facilitate the computation of the moments of the model random parameters from the experimental or field data by using the generalized series given in Equation 3.23.

3.3 IDENTIFICATION OF THE DISTRIBUTIONS OF THE MODEL RANDOM PARAMETERS

The technique developed in the preceding section estimates a set of moments for each of a_0, a_1 ... a_n. This section describes the procedure for establishing the criteria by which the distributions of model parameters can be identified from their computed, or estimated, moments.

It is known that each distribution function is identified by a set of moments when they exist [30]. The procedures by which the distribution fucntions of the model parameters a_0, a_1 ... a_n can be identified from their moments are discussed below.

3.3.1 Matching the Theoretical Distribution Moments With the Computed Moments:

In order to identify the adequate distribution of a certain model parameter, its computed moments from the modeling technique are compared with the theoretical moments of candidate distributions such as those listed in Table 3.1. Since the theoretical moments of the candidate distributions are function of the corresponding distribution parameters, the parameters of the adequate distribution of a model parameter are obtained from the computed moments of those model parameters. For example, if the distribution has two parameters, their values are obtained from the first two computed moments and the higher order theoretical moments are consequently estimated from the calculated distribution parameters. An acceptance criterion for selecting an adequate distribution from several candiate distributions is based on choosing the distribution which gives a minimum percent deviation of the computed moments from the theoretical moments.

The limitation of this approach can be summarized as follows:

a) Computed moments may include errors from the least square estimation technique. This causes errors to arise in the estimation of the distribution parameters.

b) Errors arise from computing the higher order theoretical moments from the distribution parameters, which may have errors due to (a).

As an example, for the standard normal distribution the first moment about origin and second moments about mean determine the mean μ and the variance σ^2 respectively.

From Table (3.1) the rth moment about the mean of the normal distribution is:

$$\mu_{2r} = \frac{(2r)!}{(2^r)r!}(\sigma)^{2r} \tag{3.25}$$

where

$$\mu_2 = \sigma^2.$$

After determining μ and σ^2 from the first and second moments of a model parameter, the higher order moments are computed from Equation 3.25. For example the fourth moment is given by:

$$\mu_4 = 3\sigma^4$$

or

$$\mu_4 = 3(\mu_2)^2 \tag{3.26}$$

To find the error $d\mu_4$ in μ_4 when an error $d\mu_2$ occurs in μ_2, Equation 3.26 is differentiated with respect to μ_2

to give:

$$d\mu_4 = 6\mu_2 \, d\mu_2 .$$

Therefore, any error $d\mu_2$ encountered in the determination of μ_2 will result in an error $d\mu_4$, equal to $6\mu_2$ times $d\mu_2$, in μ_4.

Defining $\frac{d\mu_i}{\mu_i}$ as the relative error in computing μ_i we get using the above equation and Equation (3.26) that

$$\frac{d\mu_4}{\mu_4} = 2 \frac{d\mu_2}{\mu_2} .$$

Again the error in computing μ_4 is twice as much as that of μ_2.

To avoid the preceding drawbacks of matching the theoretical moments with the computed moments, another approach for determining the distribution of the model parameters is proposed below.

3.3.2 Moment Ratios Method:

In this approach the distributions of the model parameters are identified without computing the distribution parameters as explained in the previous method. The proposed method uses the theoretical expression of the rth moment of the distributions, listed in Table 3.1, to develop characteristic dimensionless numbers from the moment ratios of the distributions. As an example, for the exponential distribution the rth moment about origin is given by:

$$\mu'_r = \frac{r!}{\lambda^r} \qquad r = 1, 2, \ldots$$

$$\therefore \quad \mu'_1 = \frac{1}{\lambda}$$

and

$$\frac{\mu'_r}{\mu'_{r-1}} = \frac{r}{\lambda}$$

$$\frac{\mu'_r}{\mu'_1 \; \mu'_{r-1}} = r \qquad (\text{as } \frac{1}{\lambda} = \mu'_1).$$

For $r = 2$

$$\frac{\mu'_2}{(\mu'_1)^2} = 2.$$

For $r = 3$

$$\frac{\mu'_3}{\mu'_1 \; \mu'_2} = 3, \quad \frac{\mu'_3}{\mu'_1 \cdot 2(\mu'_1)^2} = 3$$

$$\frac{\mu'_3}{(\mu'_1)^3} = 3\times 2.$$

Therefore, the general expression of the moment ratios for the exponential distribution in terms of the moment order r is given by the dimensionless number:

$$\frac{\mu_r'}{(\mu_1')^r} = r! \qquad (\text{for } \mu_1' \neq 0).$$

In a similar way, dimensionless numbers in terms of theoretical moment ratios are derived from the continuous distributions listed in Table 3.3

In order to obtain the distribution of a model parameter, the computed moments of the parameter are used to compute the actual moment ratios using the expressions of the moment ratios of the candidate distributions listed in Table 3.3. The computed moment ratios of each distribution are thus compared with the theoretical moment ratios given in Table 3.3. The acceptance criteria for an adequate distribution to represent the model parameter is based on choosing the distribution which gives a minimum percent deviation of the actual moment ratios from the theoretical moment ratios of Table 3.3. To make the computation easy, a computer program is prepared which estimates the moment ratios from the computed parameter moments and the percent deviation of the computed moment ratios from theoretical moment ratios.

Since error analysis has shown that this technique suffers to a lesser degree from the drawbacks of the moments matching techqniue, it will be used for identifying the distribution of the model parameters.

TABLE 3.3 *Theoretical Moment Ratios for Some Continuous Distributions*

No.	Distribution	Moment Ratios	r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
1.	Normal	$\frac{\mu_{2r}}{(\mu_2)^r}$ = product of first r odd number	1	3	15	105	945	10395	135135	2027025
2.	Lognormal	$\frac{\log_e[\frac{\mu'_r}{(\mu'_1)^r}]}{\log_e[\frac{\mu'_2}{(\mu'_1)^2}]} = 0.5r(r-1)$	0	1	3	6	10	15	21	28
3.	Uniform	$\frac{\mu_{2r}}{(\mu_2)^r} = \prod_{i=2}^{r} \frac{3(2i-1)}{(2i+1)}$	1	1.8	3.85	9	22.09	56.07	145.8	385.94
4.	Exponential	$\frac{\mu'_r}{(\mu'_1)^r} = r!$	1	2	6	24	120	720	5040	40320
5.	Chi Square	$\frac{\mu'_r}{\mu'_{r-1}} - \mu'_1 = 2(r-1)$	0	2	4	6	8	10	12	14
6.	Gamma	$\frac{\mu'_r\mu'_1 - \mu'^2_1\mu'_{r-1}}{\mu'_{r-1}(\mu'_2 - \mu'^2_1)} = r-1$	0	1	2	3	4	5	6	7
7.	Beta	$\frac{\frac{\mu'_2-\mu'_1}{\mu'^2_1-\mu'_2}[\frac{\mu'_r}{\mu'_{r-1}} - \mu'_1]}{[1 - \frac{\mu'_r}{\mu'_{r-1}}]} = r-1$	0	1	2	3	4	5	6	7

μ'_r = rth moment about origin μ_r = rth moment about mean

3.4 DEVELOPMENT OF A COMPUTER PROGRAM FOR THE PROPOSED STATISTICAL MODELING TECHNIQUE

To simplify the implementation of the proposed statistical modeling technique, it was appropriate to prepare a computer program, to compute the set of moments of the parameter of the following three process variables model:

$$R = KZ_1^{a_1} Z_2^{a_2} Z_3^{a_3} \tag{3.27}$$

taking the logarithm of both sides of the above equation, the following form of the model is obtained:

$$\ell n\ R = \ell n\ K + a_1 \ell n\ Z_1 + a_2 \ell n\ Z_2 + a_3 \ell n\ Z_3$$

or

$$Y = a_0 + a_1 X_1 + a_2 X_2 + a_3 X_3 \tag{3.28}$$

where

$Y = \ell n\ R$ is the dependent variable, and

$X_i = \ell n\ Z_i$ are the independent variables of the model $(i = 1, 2, 3)$.

The general expression of the rth moment of Y as a function of the process variables X_1, X_2 and X_3 and the moments of the random parameters a_0, a_1, a_2 and a_3 obtained from Equation 3.23 is:

$$\mu'_{rY} = [\{\sum_{i=0}^{r}\sum_{j=0}^{r}\sum_{k=0}^{r}\sum_{m}^{r}(\frac{r!}{i!\ j!\ k!\ m!})\}$$

$$\times \{(\mu'_{ia_0})\ (\mu'_{ja_1}X_1^j)\ (\mu'_{ka}\ X_2^k)\ (\mu'_{ma}\ X_3^m)\}] \qquad (3.29)$$

where

$r = 1, 2, \ldots$

$m = r - i - j - k \geq 0.$

The proposed computer program computes the r moments of a_0, a_1, a_2 and a_3 from the moments of the response Y measured at various sets of X_1, X_2, and X_3. A flow diagram of the program is shown in Fig. 3.1, and the procedure adopted is explained below.

a. Purpose:

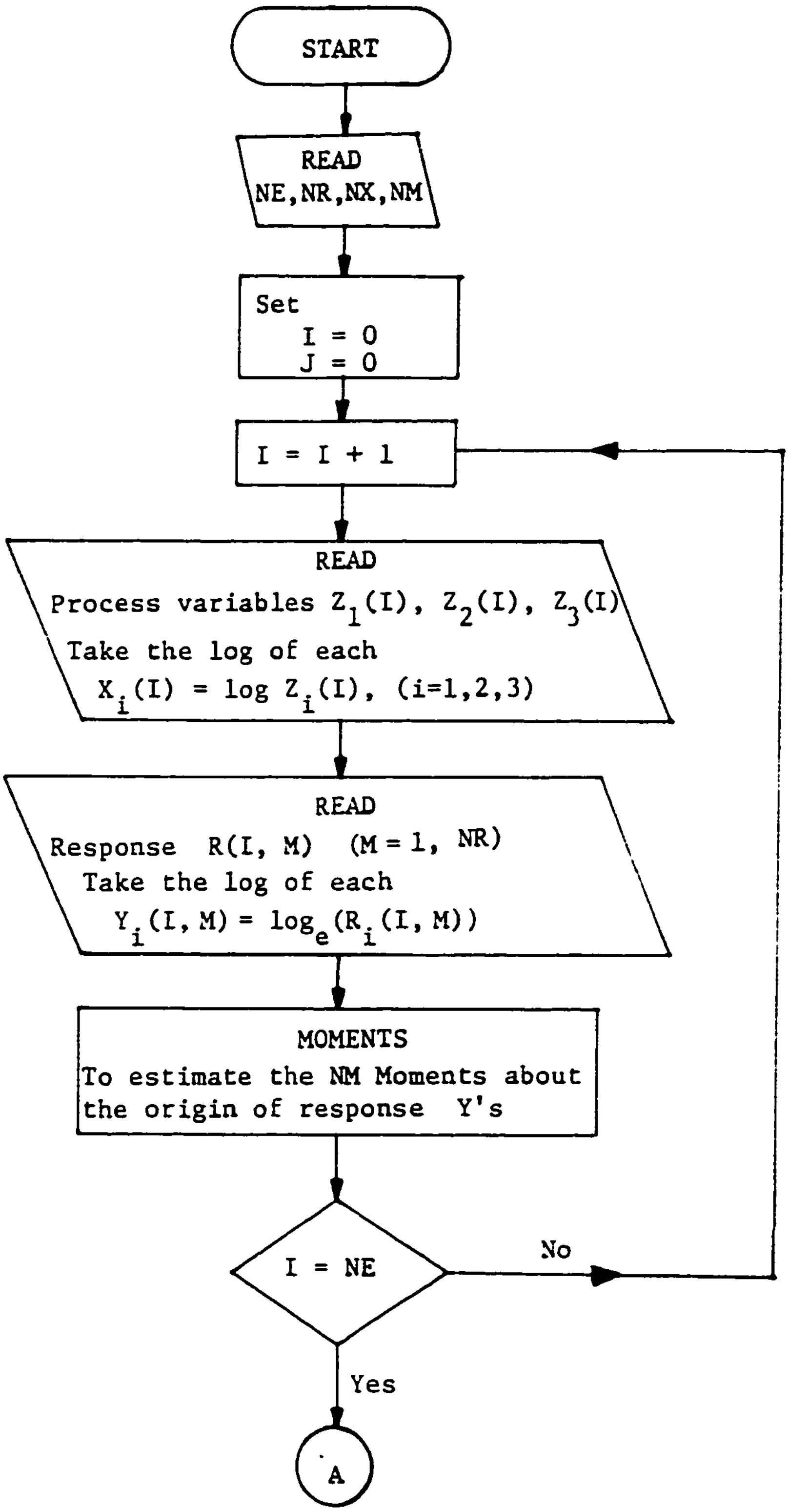
The program computes r moments of each random parameter a_0, a_1, a_2 and a_3 of the model expressed by Equation 3.28, $(r = 1, 2, \ldots)$.

b. Description of Parameters:

NE Number of data points (Experimental Sets)

NX Number of independent variables (NX = 3)

$\tilde{X}$ Matrix of the independent variables (Table 3.2)



START
READ
NE,NR,NX,NM
Set
I = 0
J = 0
I = I + 1
READ
Process variables $Z_1(I)$, $Z_2(I)$, $Z_3(I)$
Take the log of each
$X_i(I) = \log Z_i(I)$, $(i=1,2,3)$
READ
Response R(I, M) (M = 1, NR)
Take the log of each
$Y_i(I, M) = \log_e(R_i(I, M))$
MOMENTS
To estimate the NM Moments about the origin of response Y's
I = NE
No
Yes
A

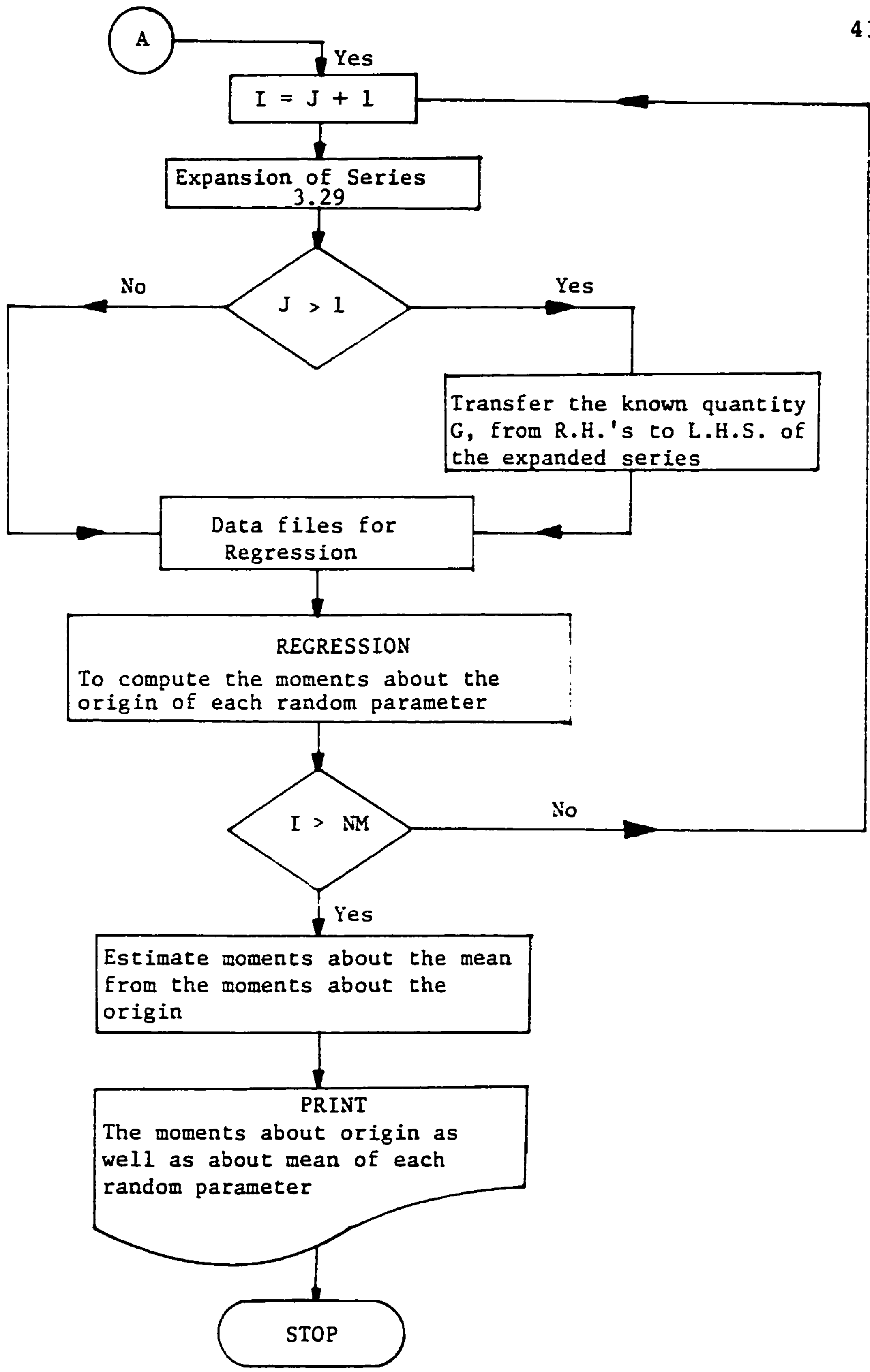


Figure 3.1 Flow Diagram of the Computer Program

NR Number of the responses measured at each experimental test condition

$\tilde{Y}$ Matrix of the responses Y (Table 3.2)

NM Number of moments

$\widetilde{UY}$ Matrix of the moments of responses Y's (Table 3.2).

c. Subroutines Required:

SUBROUTINE MOMENTS called from the main program to compute the NM moments of the NR responses of Y measured at each experimental test conditions as shown by the matrix $\widetilde{UY}$ in Table 3.2

SUBROUTINE REGREN called from the main program to compute the moments about origin of a_0, a_1, a_2 and a_3.

d. Procedure:

1. Data consisting of NR values of response R are measured at each of the NE test conditions. The values of Y_i's and X_i's are obtained by taking the natural logarithm of R ($y_i = \ell n(R_i)$ $i = 1, 2, \ldots,$ NR), and of Z_i's ($X_i = \ell n(Z_i)$, $i = 1, 2, 3$) respectively.

2. The NM moments of the NR responses of Y are computed at each test condition from the matrix $\tilde{UY}$.

3. The series of Equation 3.29 is expanded.

4. For the second, third ... rth moment of a_0, a_1, a_2 and a_3 the known quantities, denoted G_r in Equation 3.24, are transferred from the right hand side to the left hand side of the expanded series.

5. Data files having matrix $\tilde{X}^i$ and the ith column of matrix $\tilde{UY}$ are prepared to compute the ith moments of each a_0, a_1, a_2 and a_3 ($i = 1, 2 \ldots$ rth moment) using the ordinary least square estimation technique.

6. The moments about the origin of a_0, a_1, a_2 and a_3 are computed by subroutine REGREN using the data prepared in Step 5.

7. Steps 3 to 6 are repeated for computing the second, third ... rth moments about the origin of each a_0, a_1, a_2 and a_3.

8. The moments about the mean μ_r of a_0, a_1, a_2 and a_3 are estimated from their moments about the origin μ'_r by using Equation 3.11(b).

9. The distributions of a_0, a_1, a_2 and a_3 are identified from their computed moments.

e. Program Output:

The main program prints the NM moments about the origin as well as about the mean of the model parameters a_0, a_1, a_2 and a_3 respectively. The computed parameters are later supplied to another computer program which identifies the adequate distribution, out of several candidate distributions, which represents each model parameter using the moment ratios criterion developed in Section 3.2.3. Once the distribution corresponding to a_0, a_1, a_2 and a_3 is identified, the parameters of that distribution are computed from the computed moments using the rth moment expression listed in Table 3.1.

3.5 VERIFICATION OF THE DEVELOPED STATISTICAL MODELING TECHNIQUE

The previous sections describe the procedure for computing the set of moments of each model random parameter and the method of identifying the distribution of the parameter from the respective set of moments. The objective of this section is to verify the developed statistical modeling technique by proposing a model of three process variables as given by Equation 3.28. The step-by-step procedure involved in the verification process is as follows:

1. A model represented by Equation (3.28) which is a linear form of Equation (3.27) is assumed.

2. 27 test runs are assumed to be run at the test conditions listed in Table 3.4 representing a 2^3 factorial design experiment, including a 9th intermediate run, replicated three times.

3. A distribution and the related distribution parameters is assumed for each model parameter a_0, a_1, a_2 and a_3. Two types of models denoted by A and B were used in the verification. The parameters of model A are assumed to have the same type of distributions whereas the parameters of model B are not of the same distribution.

4. Using Monte-Carlo simulation, 5000 values for each of a_0, a_1, a_2 and a_3 are generated from their respective assumed probability distributions. At each of the test conditions, given in Table 3.4, 5000 values of Y are generated by using the 5000 values of a_0, a_1, a_2 and a_3. This results in 27 sets of Y each having 5000 values evaluated at each one of the 27 test conditions.

TABLE 3.4 Design Matrix of the Test Conditions

Test No.	X_1	X_2	X_3
01	0.552	0.621	0.662
02	0.552	0.621	0.621
03	0.552	0.621	0.552
04	0.552	0.552	0.662
05	0.552	0.552	0.621
06	0.552	0.552	0.552
07	0.552	0.483	0.662
08	0.552	0.483	0.621
09	0.552	0.483	0.552
10	0.516	0.621	0.662
11	0.516	0.621	0.621
12	0.516	0.621	0.552
13	0.516	0.552	0.662
14	0.516	0.552	0.621
15	0.516	0.552	0.552
16	0.516	0.483	0.662
17	0.516	0.483	0.621
18	0.516	0.483	0.552
19	0.460	0.621	0.662
20	0.460	0.621	0.621
21	0.460	0.621	0.552
22	0.460	0.552	0.662
23	0.460	0.552	0.621
24	0.460	0.552	0.552
25	0.460	0.483	0.662
26	0.460	0.483	0.621
27	0.460	0.483	0.552

5. The 27 sets of the 5000 values of Y and the sets of independent variables given in Table 3.4 are fed to the computer program developed in Section 3.4 to compute a set of moments for each model parameter a_0, a_1, a_2 and a_3. In the present study the first 8 moments are arbitrarily selected to be computed for each of the model parameters.

6. The computed moments of Step 5 above are used to identify the type of distributions of each model parameter using the moment ratios criterion proposed in Section 3.3.

7. Once the distribution of a parameter is identified, the parameters corresponding to its distribution are computed. The method of obtaining the distribution parameters from the moments is explained in Section 5.2.3.

8. The resulting distributions of Step 6 are compared with the known, assumed, distribution of Step 3.

9. During the simulation process, the moments of each model parameters are estimated from the respective generated data, then the mean and variance are obtained from the generated moments and named the generated mean and generated variance. Similarly, the mean and variance of the respective model parameters are

estimated from the computed moments (Step 5) and named the computed mean and computed variance.

10. The resulting computed mean and variance of each model are compared with its generated mean and variance to evaluate the capability of the new statistical modeling technique.

It is important to note here that the procedure of the technique verification, discussed above, was applied for two different types of models. One model (Model-A) is assumed to have all normally distributed parameters. The parameters of the second model (Model-B) used in the verification are assumed to have normal, lognormal, normal and uniform distributions for a_0, a_1, a_2 and a_3 respectively. The assumed distributions and related parameters (mean and variance) for both models are presented in Table 3.5.

The verification steps 1 to 5, described above, were used to compute the moments of the model parameters a_0, a_1, a_2 and a_3 for both models A and B listed in Table 3.6 and 3.7 respectively. The moment ratios tables required for identifying the adequate type of distribution of the model parameters are listed in Table 3.8 to 3.11 for a_0, a_1, a_2 and a_3 of model A and in Table 3.12 to 3.15 for a_0, a_1, a_2 and a_3 of model B. Each table contains the theoretical and the computed moment ratios, and the corresponding percent

TABLE 3.5 The Assumed Distributions of Models A and B Random Parameters

Model	Random Parameter	Distribution	Mean μ	Variance σ^2
	a_0	Normal	13.679	1.9650
A	a_1	Normal	- 0.366	0.0289
	a_2	Normal	1.258	0.4860
	a_3	Normal	0.278	0.0185
	a_0	Normal	11.575	3.2630
B	a_1	Lognormal	7.725	50.7013
	a_2	Normal	- 7.568	2.3510
	a_3	Uniform	3.000	0.3333

TABLE 3.6 The Computed Moments of Model-A Random Parameters

Random Variable	Moments	r=1	r=2	r=3	r=4	r=5	r=6	r =7	r=8
a_0	μ'_r	0.137×10^{2}	0.191×10^{3}	0.268×10^{4}	0.380×10^{5}	0.543×10^{6}	0.785×10^{7}	0.114×10^{8}	0.168×10^{10}
	μ_r	0.000×10^{0}	0.195×10^{1}	0.170×10^{0}	0.119×10^{2}	-0.99×10^{-2}	0.118×10^{3}	-0.65×10^{-2}	0.164×10^{4}
a_1	μ'_r	-0.378×10^{0}	0.169×10^{0}	-0.79×10^{-1}	0.37×10^{-1}	-0.18×10^{-1}	0.10×10^{-1}	-0.52×10^{-2}	0.24×10^{-2}
	μ_r	0.000×10^{0}	0.30×10^{-1}	0.63×10^{-2}	0.26×10^{-2}	-0.33×10^{-3}	0.40×10^{-3}	-0.99×10^{-3}	0.87×10^{-4}
a_2	μ'_r	0.125×10^{1}	0.205×10^{1}	0.378×10^{1}	0.768×10^{1}	0.168×10^{2}	0.397×10^{2}	0.999×10^{2}	0.266×10^{3}
	μ_r	0.000×10^{0}	0.491×10^{0}	-0.17×10^{-1}	0.705×10^{0}	-0.53×10^{-3}	0.188×10^{1}	-0.16×10^{-2}	0.624×10^{1}
a_3	μ'_r	0.288×10^{0}	0.103×10^{0}	0.43×10^{-1}	0.20×10^{-1}	0.15×10^{-1}	0.14×10^{-1}	0.19×10^{-1}	0.27×10^{-1}
	μ_r	0.000×10^{0}	0.19×10^{-1}	0.22×10^{-2}	0.12×10^{-2}	0.49×10^{-2}	0.12×10^{-3}	0.81×10^{-3}	0.16×10^{-4}

μ_r = rth moment about mean μ'_r = rth momemt about origin

TABLE 3.7 The Computed Moments of Model-B Random Parameters

Random Variable	Moments	r=1	r=2	r=3	r=4	r=5	r=6	r =7	r=8
a_0	μ'_r	0.117×10^{2}	0.139×10^{3}	0.171×10^{4}	0.214×10^{5}	0.272×10^{6}	0.354×10^{7}	0.467×10^{8}	0.628×10^{9}
	μ_r	0.000×10^{0}	0.338×10^{1}	-0.88×10^{-1}	0.329×10^{2}	-0.99×10^{-1}	0.547×10^{3}	-0.874×10^{0}	0.128×10^{6}
a_1	μ'_r	0.770×10^{1}	0.106×10^{3}	0.253×10^{4}	0.990×10^{5}	0.564×10^{7}	0.407×10^{9}	0.34×10^{11}	0.31×10^{13}
	μ_r	0.000×10^{0}	0.466×10^{2}	0.993×10^{3}	0.483×10^{5}	0.295×10^{7}	0.216×10^{9}	0.18×10^{11}	0.15×10^{13}
a_2	μ'_r	-0.754×10^{1}	0.591×10^{2}	-0.480×10^{3}	0.402×10^{4}	-0.346×10^{5}	0.306×10^{6}	-0.278×10^{7}	0.257×10^{8}
	μ_r	0.000×10^{0}	0.224×10^{1}	-0.38×10^{-2}	0.147×10^{2}	-0.13×10^{-1}	0.158×10^{3}	-0.53×10^{-1}	0.257×10^{4}
a_3	μ'_r	0.299×10^{1}	0.930×10^{1}	0.298×10^{2}	0.986×10^{2}	0.333×10^{3}	0.115×10^{4}	0.403×10^{4}	0.143×10^{5}
	μ_r	0.000×10^{0}	0.331×10^{0}	-0.11×10^{-2}	0.197×10^{0}	-0.29×10^{-2}	0.141×10^{0}	-0.76×10^{-3}	0.113×10^{0}

μ_r = rth moment about mean $\qquad$ μ'_r = rth momemt about origin

TABLE 3.8 Moment Ratios of the Theoretical and Computed Moments for a_0 of Model -A.

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.13	15.93	113.01				
		c	0.0	4.36	6.22	7.03				
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$ $/\ln(\mu'_2/\mu'^2_1)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.96	5.86	9.68	14.41	20.02	26.49
		c	0.00	0.00	1.33	2.33	3.20	3.93	4.67	5.36
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.13	15.93	113.01				
		c	0.00	73.94	313.84	1155.62				
Exponential	$\mu'_r/(\mu'_1)^r$	a	1.00	2.00	6.00	24.00	120.0	720.0	5040.0	40320.0
		b	1.00	1.01	1.03	1.06	1.1	1.2	1.2	1.3
		c	0.00	49.50	82.80	95.57	99.1	99.8	99.9	99.9
Chi-square	$\mu'_r/\mu'_{r-1} - \mu'_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.14	0.28	0.42	0.56	0.69	0.83	0.95
		c	0.00	92.80	92.92	92.98	93.03	93.07	93.11	93.152
Gamma	$\mu'_r\mu'_1-\mu'^2_1\mu'_{r-1}$ $/\mu'_{r-1}(\mu'_2-\mu'^2_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.97	2.93	3.87	4.82	5.75	6.66
		c	0.00	0.00	1.50	2.33	3.25	3.60	4.17	4.85
Beta	$(\mu'_2-\mu'_1)(\mu'_r/\mu'_{r-1} - \mu'_1)$ $/(\mu'^2_1-\mu'_2)(1-\mu'_r/\mu'_{r-1})$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.91	2.81	3.66	4.52	5.36	6.17
		c	0.00	0.00	4.50	6.33	8.55	9.54	10.66	11.87

a= Theoretical b= Computed c= % Deviation

TABLE 3.9 *Moment Ratios of the Theoretical and Computed Moments for a_1 of Model -A.*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	2.95	14.89	106.67				
		c	0.0	1.66	0.70	1.59				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	b	0.00	1.00	2.16	3.40	4.85	6.75	8.41	9.54
	$/\ln(\mu_2'/\mu_1'^2)$	c	0.00	0.00	27.86	43.28	51.48	55.03	59.96	65.91
		a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}/(\mu_2)^r$	b	1.00	2.95	14.89	106.67				
		c	0.00	63.88	286.88	1085.31				
		a	1.00	2.00	6.00	24.00	120.0	720.0	5040.0	40320.0
Exponential	$\mu_r'/(\mu_1')^r$	b	1.00	1.22	1.53	1.95	2.6	3.7	5.2	6.5
		c	0.00	39.20	74.55	91.88	97.8	99.4	99.9	99.9
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	b	0.00	-0.08	-0.09	-0.10	-0.12	-0.17	-0.14	-0.09
		c								
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.18	1.27	1.52	2.07	1.78	1.15
	$/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	c	0.00	0.00	40.85	57.70	62.07	58.48	70.38	83.54
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.17	1.25	1.47	1.96	1.70	1.14
	$/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	c	0.00	0.00	41.40	58.33	63.12	60.82	71.60	83.68

a= Theoretical b= Computed c= % Deviation

TABLE 3.10 *Moment Ratios of the Theoretical and Computed Moments for a_2 of Model -A.*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	2.93	15.94	109.82				
		c	0.0	2.33	6.28	4.59				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$ $/\ln(\mu_2'/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.41	4.19	6.24	8.56	11.13	13.90
		c	0.00	0.00	19.50	30.15	37.54	42.87	46.95	50.35
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	2.93	15.94	109.82				
		c	0.00	62.77	314.10	1120.20				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.0	720.0	5040.0	40320.0
		b	1.00	1.31	1.93	3.13	5.5	10.3	20.7	44.2
		c	0.00	34.35	67.82	86.94	95.4	98.6	99.6	99.9
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.39	0.59	0.78	0.94	1.12	1.26	1.41
		c	0.00	80.40	85.28	87.01	88.26	88.94	89.45	89.93
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$ $/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.50	1.99	2.40	2.82	3.23	3.60
		c	0.00	0.00	24.90	33.80	40.07	43.58	46.22	48.61
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$ $/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.15	1.24	1.29	1.34	1.37	1.39
		c	0.00	0.00	42.50	58.63	67.62	73.26	77.18	80.10

a= Theoretical b= Computed c= % Deviation

TABLE 3.11 Moment Ratios of the Theoretical and Computed Moments for a_3 of Model -A.

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.19	16.11	111.35				
		c	0.0	6.43	7.41	5.05				
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.75	5.05	12.05	18.98	30.66	39.52
	$/\ln(\mu'_2/\mu_1'^2)$	c	0.00	0.00	8.50	15.78	20.55	26.53	46.00	41.14
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.19	16.11	111.35				
		c	0.00	7.73	318.00	1137.20				
Exponential	$\mu'_r/(\mu'_1)^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.23	1.79	2.29	12.80	55.90	664.70	4341.78
		c	0.00	38.20	70.16	87.83	89.27	92.23	86.82	89.23
Chi-square	$\mu'_r/\mu'_{r-1} - \mu'_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.07	0.13	0.18	0.98	0.96	3.14	1.59
		c	0.00	96.60	96.75	96.97	87.71	90.40	73.86	88.64
Gamma	$\mu'_r\mu'_1 - \mu_1'^2\mu'_{r-1}$	a	0.00	1.00	2.00	3.00	8.00	5.00	6.00	7.00
		b	0.00	1.00	1.89	2.67	14.44	14.16	46.11	23.43
	$/\mu'_{r-1}(\mu'_2-\mu_1'^2)$	c	0.00	0.00	5.5	11.00	261.00	152.68	668.40	2.34
Beta	$(\mu'_2-\mu'_1)(\mu'_r/\mu'_{r-1} - \mu'_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	2.09	3.25	-34.34	-36.26	-12.24	-17.11
	$/(\mu_1'^2-\mu'_2)(1-\mu'_r/\mu'_{r-1})$	c	0.00	0.00	4.50	8.33	959.50	825.20	304.00	344.40

a= Theoretical   b= Computed   c= % Deviation

TABLE 3.12 Moment Ratios of the Theoretical and Computed Moments for a_0 of Model -B.

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	2.87	14.11	97.87				
		c	0.0	4.10	5.88	6.78				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	b	0.00	1.00	2.92	5.71	9.31	13.66	18.73	24.47
		c	0.00	0.00	2.43	4.71	6.87	8.90	10.82	12.62
	$/\ln(\mu_2'/\mu_1'^2)$									
		a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}'/(\mu_2')^r$	b	1.00	2.87	14.11	97.87				
		c	0.00	59.83	266.67	987.46				
		a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
Exponential	$\mu_r'/(\mu_1')^r$	b	1.00	1.02	1.07	1.15	1.25	1.39	1.58	1.82
		c	0.00	48.75	82.10	95.21	98.95	99.81	99.97	99.99
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	b	0.00	0.29	0.56	0.82	1.08	1.32	1.54	1.76
		c	0.00	85.50	85.87	86.21	86.54	86.84	87.12	87.39
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$	a	0.00	1.00	2.00	3.00	8.00	5.00	6.00	7.00
		b	0.00	1.00	1.94	2.85	3.71	4.53	5.32	6.09
	$/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	c	0.00	0.00	2.50	4.93	7.17	9.28	11.23	13.04
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.90	2.79	3.47	4.15	4.78	5.36
	$/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	c	0.00	0.00	4.90	9.37	13.40	17.04	20.35	23.35

a= Theoretical b= Computed c= % Deviation

TABLE 3.13 Moment Ratios of the Theoretical and Computed Moments for a_1 of Model -B.

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	22.20	2131.79	322396.8				
		c	0.00	640.00	14111.9	306943.8				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$	b	0.00	1.00	2.95	5.75	9.21	13.06	17.16	21.39
	$/\ln(\mu'_2/\mu_1'^2)$	c	0.00	0.00	1.71	4.10	7.99	12.95	18.31	23.58
		a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}/(\mu_2)^r$	b	1.00	22.20	2131.79	322396.8				
		c	0.00	1133.3	55271.4	3582078.				
		a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
Exponential	$\mu'_r/(\mu'_1)^r$	b	1.00	1.78	5.53	28.16	208.39	1954.60	21083.00	247354.50
		c	0.00	10.71	7.75	17.32	73.65	171.47	318.31	513.47
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu'_r/\mu'_{r-1} - \mu'_1$	b	0.00	6.06	16.15	31.47	42.28	64.52	75.36	79.64
		c	0.00	202.85	303.81	416.67	436.00	545.36	527.99	590.30
Gamma	$\mu'_r\mu'_1-\mu_1'^2\mu'_{r-1}$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	2.67	5.19	8.14	10.65	12.44	13.64
	$/\mu'_{r-1}(\mu'_2-\mu_1'^2)$	c	0.00	0.00	33.35	73.20	103.51	113.00	107.30	94.85
Beta	$(\mu'_2-\mu'_1)(\mu'_r/\mu'_{r-1} - \mu'_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.49	1.73	1.85	1.91	1.93	1.95
	$/(\mu_1'^2-\mu'_2)(1-\mu'_r/\mu'_{r-1})$	c	0.00	0.00	25.60	42.13	53.75	61.80	67.76	72.17

a= Theoretical b= Computed c= % Deviation

TABLE 3.14 Moment Ratios of the Theoretical and Computed Moments for a_2 of Model -B.

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	2.92	13.99	101.79				
		c	0.00	2.71	6.72	3.05				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$	b	0.00	1.00	2.89	5.58	9.01	13.12	17.84	23.14
	$/\ln(\mu'_2/\mu_1'^2)$	c	0.00	0.00	3.63	6.88	9.85	12.56	15.06	17.36
		a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}/(\mu_2)^r$	b	1.00	2.92	13.99	101.79				
		c	0.00	62.11	263.42	1031.06				
		a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
Exponential	$\mu'_r/(\mu'_1)^r$	b	1.00	1.04	1.12	1.24	1.42	1.66	1.99	2.44
		c	0.00	48.05	81.36	94.83	98.82	99.77	99.96	99.99
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu'_r/\mu'_{r-1} - \mu'_1$	b	0.00	-0.29	-0.57	-0.83	-1.07	-1.29	-1.51	-1.71
		c	0.00	114.81	114.31	113.81	113.35	112.95	112.50	112.24
Gamma	$\mu'_r\mu'_1 - \mu_1'^2\mu'_{r-1}$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.92	2.78	3.59	4.36	5.08	5.77
	$/\mu'_{r-1}(\mu'_2 - \mu_1'^2)$	c	0.00	0.00	3.80	7.13	10.12	12.84	15.31	17.61
Beta	$(\mu'_2 - \mu'_1)(\mu'_r/\mu'_{r-1} - \mu'_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.86	2.63	3.31	3.92	4.47	4.97
	$/(\mu_1'^2 - \mu'_2)(1 - \mu'_r/\mu'_{r-1})$	c	0.00	0.00	6.71	12.41	17.32	21.68	25.53	28.97

a= Theoretical b= Computed c= % Deviation

TABLE 3.15 *Moment Ratios of the Theoretical and Computed Moments for a_3 of Model -B.*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	1.81	3.91	9.45				
		c	0.0	39.76	73.92	90.99				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$ $/\ln(\mu_2'/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.89	5.57	8.18	12.26	17.12	21.85
		c	0.00	0.00	3.56	8.35	12.19	14.77	18.45	21.98
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	1.81	3.91	9.45				
		c	0.00	0.30	3.60	5.04				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.04	1.11	1.22	1.38	1.59	1.86	2.21
		c	0.00	48.15	81.48	94.67	94.91	95.20	95.75	96.01
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.11	0.21	0.31	0.38	0.45	0.51	0.56
		c	0.00	94.50	94.67	94.91	95.20	95.47	95.75	96.01
Gamma	$\mu_r'\mu_1'-\mu_1'^2\mu_{r-1}'$ $/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.93	2.76	3.48	4.10	4.62	5.05
		c	0.00	0.00	3.70	8.13	13.05	18.10	23.06	27.80
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$ $/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.84	2.52	3.08	3.52	3.88	4.16
		c	0.00	0.00	8.15	15.90	23.05	29.54	35.33	40.47

a= Theoretical b= Computed c= % Deviation

deviation. The percent deviation between the corresponding theoretical and computed ratios is defined as:

$$\text{Percent deviation} = \left(\frac{\text{Theoretical} - \text{Computed}}{\text{Theoretical}}\right) \times 100$$

Table 3.16 lists the generated and the computed means and variances (Step 9) of the model parameters a_0, a_1, a_2 and a_3 for model A and model B. The calculated percent deviations of the computed values from the generated values are also included in the table.

The discussion of the results obtained during the verification of the proposed technique is given in Section 3.6.

3.6 DISCUSSION OF THE RESULTS:

The objective of the verification covered in Section 3.5 was to demonstrate the use of the developed statistical modeling technique to obtain the distributions and the related distribution parameters of the parameters of a known model. In this section, the results obtained from the modeling technique are compared with the expected known results in order to check the validity of the developed modeling technique.

During the subsequent discussion reference is made to Table 3.5 to 3.16 which include the numerical results from the verification of the proposed modeling technique.

TABLE 3.16 Comparison of the Generated and Computed Means and Variances of Models A and B Parameters

Model	Parameter	Distribution	Mean Generated μ	Mean Computed $\hat{\mu}$	% Deviation $\frac{(\mu-\hat{\mu})*100}{\mu}$	Variance Generated σ^2	Variance Computed $\hat{\sigma}^2$	% Deviation $\frac{(\sigma^2-\hat{\sigma}^2)*100}{\sigma^2}$
	a_0	Normal	13.681	13.751	0.51	1.96 7	1.951	0.82
A	a_1	Normal	- 0.366	- 0.373	1.63	0.02 9	0.030	3.45
	a_2	Normal	1.263	1.252	0.87	0.48 6	0.491	1.02
	a_3	Normal	0.285	0.288	1.05	0.01 8	0.019	1.51
	a_0	Normal	11.719	11.673	0.39	3.39 5	3.385	0.29
B	a_1	Lognormal	7.687	7.700	0.17	45.08 7	46.645	3.45
	a_2	Normal	- 7.547	- 7.547	0.24	2.26 4	2.243	0.93
	a_3	Uniform	2.995	2.996	0.03	0.32 8	0.331	0.91

A closer examination of Table 3.16 will indicate that for Model-A, whose random parameters are normally distributed, the deviation of the computed mean from the generated mean ranges between 0.5 to 1.63 percent and the deviation of the computed variance from the generated variance ranges between 0.41 to 3.45 percent. As for Model-B, it is observed from the same table that the deviation of the computed mean from the generated mean ranges between 0.17 to 0.39 percent and the deviation of computed variance from the generated variance ranges between 0.29 to 3.45 percent. Since the maximum observed deviation is about 3.5 percent, it is possible to conclude that the proposed modeling technique was able to compute the means and variances of each model parameter within a reasonable percent deviation. The observed deviation between the generated and computed values are attributed to a) the errors which arise from the random number generation during the Monte-Carlo simulation and b) the errors which arise from the ordinary least square method used to estimate the moments of the model parameters a_0, a_1, a_2 and a_3.

Although it was possible to obtain the means and variances of the distributions within a reasonable percent deviation using the proposed modeling technique, an equally important goal of the verification is to identify the distributions of the model parameters and compare them with their known distributions.

a. Identification of the Distributions of the Parameters of Model-A:

As shown in Table 3.5, the random parameters of Model-A were assumed to be normally distributed. The adequate distribution to represent each model parameter will be identified by comparing the ratio of computed moments with the ratio of theoretical moments listed in Table 3.8 to 3.11 respectively. In addition to the moment ratios, the tables include the percent deviation of the computed moment ratios from the theoretical moment ratios corresponding to each candidate distribution. The criterion used for identifying the adequate distribution is based on accepting the distribution with the minimum percent deviation of the computed moment ratios from the theoretical moment ratios.

From the moment ratios table of a_0 (Table 3.8), on the basis of minimum percent deviation between the theoretical and computed moment ratios, it is observed that the distributions of a_0 can be reasonably represented by normal, lognormal or gamma distributions, inspite of the fact that the assumed distribution of a_0 was normal. This result is explained by the fact that some distributions tend to approach other distributions at certain levels of the distribution parameters. It is known that lognormal can give a good representation of the normal distribution if the value of the coefficient of variation (C.O.V.) is less than 0.25 [31], i.e., C.O.V. $= \frac{\sigma}{\mu} \leq 0.25$. Referring to the results of a_0, the C.O.V. of a_0 is equal to 0.101. Another known fact is that if the shape parameter α of gamma distribution is high as compared to its scale parameter θ (Table 3.1), then the gamma distribution approaches the normal distribution [31]. In the present case, it is found from the moments of a_0, that

$$\alpha = \frac{\mu^2}{\sigma^2} = 96.88, \text{ and } \theta = \frac{\mu}{\sigma^2} = 7.45.$$

Therefore, it is concluded from these computations that the distribution of a_0 can be represented by normal, lognormal or gamma distribution at the same time.

As for parameters a_1, a_2 and a_3 of Model-A, comparison of the theoretical and computed moment ratios of these random parameters (Table 3.9, 3.10 and 3.11) shows that, on the basis of minimum percent deviation, the distributions of all parameters are identified to be represented by a normal distribution, which are the same as their assumed distributions listed in Table 3.5.

b. Identification of the Distribution of the Parameters of Model-B:

As shown in Table 3.5, the random parameters a_0, a_1,

a_2 and a_3 for Model-B were assumed to have normal, lognormal, normal and uniform distributions respectively. The moment ratios from the computed moments of these parameters were estimated and listed in Table 3.13 to 3.16 respectively. From the moment ratios table of a_0 (Table 3.12) and on the basis of minimum percent deviation criterion of the theoretical moments from the computed moment ratios, it is observed that the distribution of a_0 can be represented by normal, lognormal or gamma distributions, in spite of the fact that the distribution of a_0 was assumed as normal. The reason for this discrepancy is again attributable to the facts which are explained in Model-A. In the present case the C.O.V. of a_0 is less than 0.25 (C.O.V. = 0.157) and the shape parameter α, of gamma (α = 40) distribution is high as compared to its scale parameter θ (θ = 3.45). Therefore, for Model-B, the parameter a_0 may again be represented by any of the three normal, lognormal or gamma distributions.

On the other hand, the random parameters a_1, a_2 and a_3 of the Model-B were assumed to be lognormally, normally and uniformly distributed. The comparison of the theoretical and computed moment ratios of these parameters (Table 3.13, 3.14 and 3.15) shows that the distributions of each parameter comes out the same as were assumed.

From the above discussion of the statistical modeling technique verification, it is observed that for a random parameter of the model, if the coefficient of variation (C.O.V.) is less than 0.24, two or more distributions (normal, lognormal, gamma, etc.) are identified by that parameter. This behavior is explained further by assuming another model (Model-C) whose parameters are normally distributed with C.O.V. which is greater than 0.24, as shown in Table 3.17. Using the same procedure of the verification as explained the Model-A and B, the set of moments computed are listed in Tabel 3.18 and the theoretical and the computed moment ratios with the corresponding percent deviations are presented in Table 3.19 to 3.22 for each parameter a_0, a_1, a_2 and a_3 respectively of Model-C.

A closer examination of the moment ratios tables (Table 3.19 to 3.22) shows that the parameters a_0, a_1, a_2 and a_3 are uniquely represented by the normal distribution as were assumed. In this case, when C.O.V. is greater than 0.24, other distributions, say lognormal or gamma, are not found to be represented by any of the random parameter of the model.

TABLE 3.17 The Assumed Distributions of Model-C Random Parameters

Random Parameter	Distribution	Mean μ	Variance σ^2	Coefficient of Variation
a_0	Normal	13.679	13.9870	0.273
a_1	Normal	- 0.366	0.0289	0.464
a_2	Normal	1.258	0.4860	0.556
a_3	Normal	0.278	0.0185	0.477

TABLE 3.18 The Computed Moments of Model-C Random Parameters

Random Variable	Moments	r=1	r=2	r=3	r=4	r=5	r=6	r =7	r=8
a_0	μ_r'	$0.133\text{x}10^{2}$	$0.193\text{x}10^{3}$	$0.295\text{x}10^{4}$	$0.473\text{x}10^{5}$	$0.796\text{x}10^{6}$	$0.139\text{x}10^{8}$	$0.253\text{x}10^{9}$	$0.476\text{x}10^{10}$
	μ_r	$0.000\text{x}10^{0}$	$0.137\text{x}10^{2}$	$-0.65\text{x}10^{-1}$	$0.589\text{x}10^{3}$	$-0.51\text{x}10^{-1}$	$0.414\text{x}10^{5}$	$-0.950\text{x}10^{0}$	$0.398\text{x}10^{7}$
a_1	μ_r'	$-0.371\text{x}10^{0}$	$0.166\text{x}10^{0}$	$-0.113\text{x}10^{0}$	$0.906\text{x}10^{0}$	$-0.129\text{x}10^{0}$	$0.183\text{x}10^{0}$	$-0.215\text{x}10^{0}$	$0.218\text{x}10^{0}$
	μ_r	$0.000\text{x}10^{0}$	$0.288\text{x}10^{0}$	$-0.30\text{x}10^{-1}$	$0.24\text{x}10^{-2}$	$-0.603\text{x}10^{-1}$	$0.37\text{x}10^{-3}$	$-0.80\text{x}10^{-2}$	$0.76\text{x}10^{-4}$
a_2	μ_r'	$0.122\text{x}10^{1}$	$0.199\text{x}10^{1}$	$0.366\text{x}10^{1}$	$0.751\text{x}10^{1}$	$0.167\text{x}10^{2}$	$0.396\text{x}10^{2}$	$0.993\text{x}10^{2}$	$0.261\text{x}10^{3}$
	μ_r	$0.000\text{x}10^{0}$	$0.488\text{x}10^{0}$	$-0.15\text{x}10^{-1}$	$0.747\text{x}10^{0}$	$-0.34\text{x}10^{-1}$	$0.183\text{x}10^{1}$	$-0.90\text{x}10^{-1}$	$0.644\text{x}10^{1}$
a_3	μ_r'	$0.283\text{x}10^{0}$	$0.100\text{x}10^{0}$	$0.42\text{x}10^{-1}$	$0.20\text{x}10^{-1}$	$0.12\text{x}10^{-1}$	$0.79\text{x}10^{-2}$	$0.61\text{x}10^{-2}$	$0.52\text{x}10^{-1}$
	μ_r	$0.000\text{x}10^{0}$	$0.19\text{x}10^{-1}$	$0.25\text{x}10^{-2}$	$0.12\text{x}10^{-2}$	$0.16\text{x}10^{-2}$	$0.13\text{x}10^{-3}$	$0.76\text{x}10^{-3}$	$0.17\text{x}10^{-4}$

μ_r = rth moment about mean μ_r' = rth momemt about origin

TABLE 3.19 Moment Ratios of the Theoretical and Computed Moments for a_0 of Model -C.

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.14	16.10	113.01				
		c	0.0	4.73	7.34	7.63				
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$ $/\ln(\mu'_2/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.80	5.29	8.39	12.05	16.20	20.80
		c	0.00	0.00	6.57	11.75	16.03	19.66	22.85	25.68
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.14	16.10	113.01				
		c	0.00	74.55	318.23	1155.67				
Exponential	$\mu'_r/(\mu'_1)^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.07	1.23	1.47	1.86	2.43	3.30	4.64
		c	0.00	46.17	79.50	93.81	98.45	99.66	99.93	99.98
Chi-square	$\mu'_r/\mu'_{r-1} - \mu'_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	1.02	1.90	2.69	3.44	4.13	4.79	5.41
		c	0.00	48.80	52.43	55.01	57.00	58.64	60.06	61.35
Gamma	$\mu'_r\mu'_1 - \mu_1'^2\mu'_{r-1}$ $/\mu'_{r-1}(\mu'_2 - \mu_1'^2)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.86	2.63	3.35	4.04	4.68	5.28
		c	0.00	0.00	7.09	12.13	16.02	19.22	22.00	24.50
Beta	$(\mu'_2 - \mu'_1)(\mu'_r/\mu'_{r-1} - \mu'_1)$ $/(\mu_1'^2 - \mu'_2)(1 - \mu'_r/\mu'_{r-1})$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.74	2.34	2.85	3.38	3.65	3.98
		c	0.00	0.00	12.80	21.84	28.84	34.44	39.11	43.12

a= Theoretical b= Computed c= % Deviation

TABLE 3.20 Moment Ratios of the Theoretical and Computed Moments for a_1 of Model -C.

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	2.93	15.55	110.95				
		c	0.0	2.49	3.66	5.67				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	4.21	8.23	15.33	22.38	28.46	33.74
	$/\ln(\mu_2'/\mu_1'^2)$	c	0.00	0.00	40.39	37.21	53.30	49 1	35.52	20.51
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	2.93	15.55	110.95				
		c	0.00	62.52	303.87	1132.83				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.21	2.22	4.77	18.34	69.99	221.55	603.81
		c	0.00	39.55	62.93	80.12	84.71	90.28	95.60	98.50
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	-0.08	-0.31	-0.42	-1.06	-1.04	-0.80	-0.64
		c	0.00	103.87	107.79	107.08	113.20	110.45	106.69	104.58
Gamma	$\mu_r'\mu_1'-\mu_1'^2\mu_{r-1}'$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	4.02	5.48	13.62	13.47	14.67	18.25
	$/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	c	0.00	0.00	100.88	82.63	240.40	169.50	172.70	178.23
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	3.46	4.42	8.13	8.08	9.90	11.90
	$/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	c	0.00	0.00	72.94	47.28	103.16	61.57	78.56	90.32

a= Theoretical b= Computed c= % Deviation

TABLE 3.21 Moment Ratios of the Theoretical and Computed Moments for a_2 of Model -C.

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	3.14	15.74	113.86				
		c	0.0	4.71	4.95	8.44				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	b	0.00	1.00	2.43	4.26	6.37	8.73	11.28	13.99
	$/\ln(\mu_2'/\mu_1'^2)$	c	0.00	0.00	18.84	28.95	36.23	41.79	46.29	50.00
		a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}'/(\mu_2')^r$	b	1.00	3.14	15.74	113.86				
		c	0.00	74.52	308.91	1165.11				
		a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
Exponential	$\mu_r'/(\mu_1')^r$	b	1.00	1.32	1.97	3.30	5.97	11.55	23.59	50.54
		c	0.00	33.82	67.02	86.24	95.02	98.39	99.53	99.87
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	b	0.00	0.39	0.61	0.82	0.99	1.14	1.28	1.41
		c	0.00	80.14	84.80	86.31	87.59	88.53	89.33	89.98
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.50	2.07	2.50	2.89	3.22	3.53
	$/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	c	0.00	0.00	23.48	31.06	37.51	42.23	46.30	49.54
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.15	1.23	1.28	1.31	1.33	1.35
	$/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	c	0.00	0.00	42.76	58.93	67.99	73.74	77.73	80.66

a= Theoretical b= Computed c= % Deviation

TABLE 3.22 Moment Ratios of the Theoretical and Computed Moments for a_3 of Model -C.

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.16	16.02	112.76				
		c	0.0	5.22	6.81	7.02				
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$ $/\ln(\mu'_2/\mu'^2_1)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.78	5.13	8.38	12.29	16.86	21.79
		c	0.00	0.00	7.22	14.47	16.19	18.03	19.71	22.15
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.16	16.02	112.76				
		c	0.00	75.36	316.14	1152.96				
Exponential	$\mu'_r/(\mu'_1)^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.25	1.85	3.12	6.40	15.23	41.89	125.06
		c	0.00	37.60	69.12	87.01	94.66	97.88	99.17	99.68
Chi-square	$\mu'_r/\mu'_{r-1} - \mu'_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.07	0.14	0.19	0.29	0.39	0.49	0.56
		c	0.00	96.50	96.57	96.78	96.27	96.09	95.87	95.98
Gamma	$\mu'_r\mu'_1 - \mu'^2_1\mu'_{r-1}$ $/\mu'_{r-1}(\mu'_2 - \mu'^2_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.95	2.75	4.25	5.56	7.05	8.00
		c	0.00	0.00	2.31	8.28	6.25	11.29	17.57	14.38
Beta	$(\mu'_2 - \mu'_1)(\mu'_r/\mu'_{r-1} - \mu'_1)$ $/(\mu'^2_1 - \mu'_2)(1 - \mu'_r/\mu'_{r-1})$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	2.18	3.39	6.57	11.04	20.62	33.55
		c	0.00	0.00	8.98	13.27	64.26	120.85	243.70	379.32

a= Theoretical b= Computed c= % Deviation

3.7 SUMMARY AND CONCLUSION

The developed statistical modeling technique treats the parameters of the model as random variables and computes their moments from the experimental or field data. These moments are then used for identifying the distributions of the model parameters as well as the related distribution parameters. The verification of the technique was performed in two steps by using the generated data of simulation. In the first step, the mean and variance of each model parameters were estimated from the computed moments and compared with the mean and variance obtained from the generated data of the respective model parameters. In the second step the distributions of each model parameter were identified from the respective set of moments and then compared with the assumed distributions.

From the verification of the statistical modeling technique, it is concluded that the developed technique computes the desired mean and variance of each model parameter within reasonable percent deviation. The deviation in the computed and generated variances is high as compared to the deviation in the computed and generated means. Similarly during the identification of the distributions, by comparing the theoretical and computed moment ratios of the respective model parameters, it indicates that in most of the cases the percent deviation increases from lower to higher order of the corresponding theoretical and computed moment ratios. This inverse of deviation in higher order moments is obvious, as the statistical modeling technique computes the higher successive order and moments by using the lower order of moments. The errors arise in the first moments will be added in the computation of second moments; this error will be added in the computation of third moments and so on.

Another important conclusion resulting from the verification results is the fact that a set of moments computed by statistical modeling technique does not necessarily identify a unique distribution. For example, if the coefficient of variation (C.O.V.) in a set of moments is less than or equal to 0.24, both the normal and the lognormal distributions are suitable for that set of moments. Similarly, if the shape parameter α of the gamma distribution is high as compared to its cale parameters α, estimated from the moments, then the normal and gamma distributions are the candidates for that set of moments.

This behavior of nonuniqueness was observed during the verification in parameter a_0 of Model-A and Model-B, where the distributions of a_0 for both models were assumed to be normally distributed with a C.O.V. less than 0.24. During the identification of a_0 distributions, normal, lognormal and gamma distributions were found to be adequate for a_0. This nonuniqueness problem was avoided in Model-C where all parameters were assumed normally distributed with C.O.V. greater than 0.24. In this case the distributions obtained for each model parameter were uniquely normal.

4. EXPERIMENTAL WORK TO GENERATE SURFACE ROUGHNESS DATA

In Chapter 5, the developed statistical modeling technique which was described in Chapter 3 will be used for developing the surface roughness models of a fine turning process. Since the technique is data dependent, this chapter describes the experimental work which has been performed; a) to machine fine turning surfaces at different sets of cutting conditions, b) to generate surface profiles from the fine turned surfaces, and c) to digitize the generated surface profiles at an optimum sampling interval.

Section 4.1 covers the experimental procedure for fine turning a mild steel shaft at different sets of cutting conditions determined by a two-level three-variable factorial design. Section 4.2 details the generation and recording of the surface profiles of the turned surfaces using a Talysurf-4 surface roughness measuring instrument and an FM-tape recorder. Section 4.3 describes the setup used for the profile data acquisition and the procedure of obtaining an optimum sampling interval for digitizing the generated profiles.

4.1 THE FINE TURNING PROCESS

This section describes the procedure for obtaining fine turned surfaces at different sets of cutting conditions.

4.1.1 The Design of Experiments

It is well known that the characteristics of surface roughness produced by fine turning depends on the various process variables such as cutting speed, feed rate, depth of cut, tool geometry, tool and workpiece material, lubrication condition etc. The present investigation will consider the effect of cutting speed, feed rate and depth of cut only by assuming the other variables as constants. A 2^3 (2-level, 3-variable) factorial design matrix was used for generating the fine turned surfaces. The design matrix for the experiment is given in Table 4.1, togehter with the selected levels of the cutting conditions [46]. In addition to the 8 test runs of the factorial design, a 9th run at the center of the design is also included. In order to provide enough data to detect the random nature of the machining process, each test was replicated three times. Therefore, a total of 27 test runs were performed for generating the surface roughness data.

4.1.2 Experimental Procedure

A 10 h.p., Colchester/Mascot 1600, medium lathe was used for the fine turning operation. The workpiece material used was an EN08, British Standard Steel. The cutting tools used were tungsten carbide disposable inserts with a 6° rake angle, $\frac{1}{32}$ inch nose radius and a 10° end relief angle. The inserts were mounted in a tool holder having a 5/8 inch square shank

Table 4.1 The Experimental Design Matrix For Test Runs.

Test Run No.	Feed Rate F(mm/rev)	Cutting Speed V(m/min)	Depth of cut D(mm)	Order of Test Runs performed		
1	0.4	215.0	0.7	10	26	1
2	0.4	215.0	0.3	7	8	3
3	0.4	100.0	0.7	13	14	20
4	0.4	100.0	0.3	15	6	27
5	0.2	215.0	0.7	17	12	9
6	0.2	215.0	0.3	24	11	2
7	0.2	100.0	0.7	25	5	4
8	0.2	100.0	0.3	19	22	18
9	0.3	160.0	0.5	16	21	23

Coding Plan	Feed rate	Cutting Speed	Depth of Cut
Low Level	0.2	100.0	0.3
Centre Level	0.3	160.0	0.5
High Level	0.4	215.0	0.7

and a 15° side cutting edge angle. The tungsten carbide insert was placed in the tool holder and a mechanical type chip-braker was placed above the insert.

In order to perform the 27 test runs, 3 bars of the workpiece, measuring 75.0 mm diameter and 419.1 mm length each were prepared. Each bar was divided into 9 sections of 38.1 mm length each, by providing 9 grooves as shown in Fig. 4.1.

After mounting each workpiece between the lathe centers a preliminary rough cut was taken on the whole length of the bar, with a feed rate of 0.1 mm/rev, a cutting speed of 118.5 m/min, a depth of cut of 0.3 mm, and a tool nose radius of 0.793 mm. The 9 sections of each bar were then machined using the randomized order of the test runs given in the last three columns of Table 4.1. All the sections were machined dry, and a new carbide insert was used for each test run. The workpiece was then removed from the lathe and was taken to the test bench for recording the machined surface profiles.

4.2 SURFACE ROUGHNESS PROFILE MEASUREMENT

In single-point machining, the surface profiles differ in the directions that are parallel and perpendicular to the

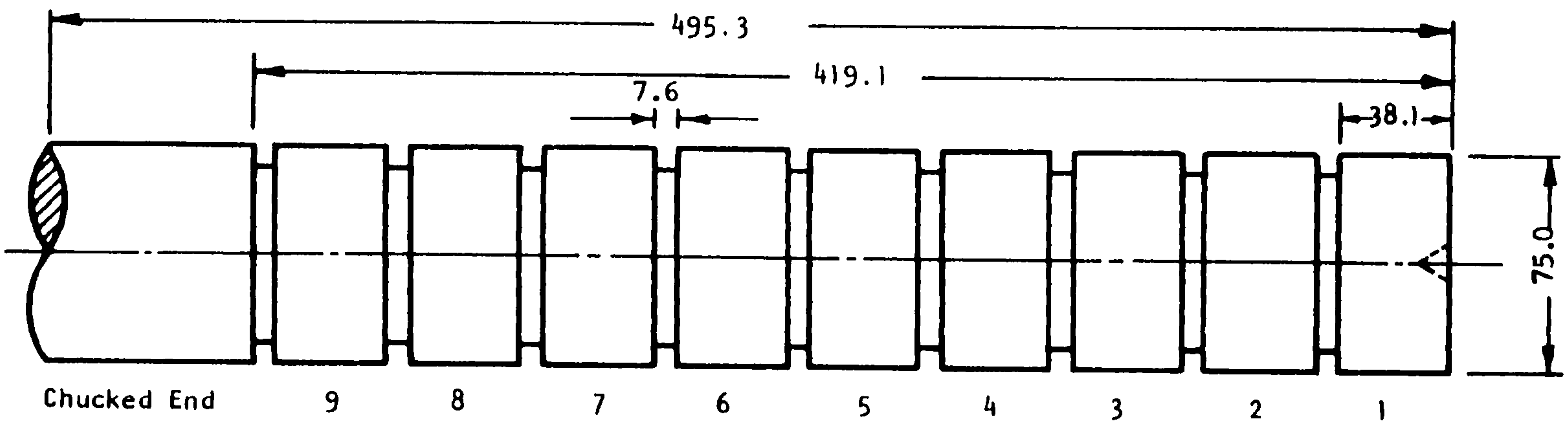


Figure 4.1 Geometry of the EN08 Steel Workpiece

tool feeding direction. Thus, in reality the surface generated by turning are anisotropic in nature, and exhibit a variety of directional characteristics. Although researchers [32-34] have developed the techniques to characterise the anisotropic surface, only the surface profiles in tool feeding directions are examined in this study because a significant amount of the geometric informations of a turned surface are in the tool feeding direction.

In single-point machining, factors such as built-up edge, chatter, tool wear, vibration etc., modify the surface profiles in a random way [24-26]. This randomness of the surface profiles in tool feeding direction cause the random values to vary, even under one specific set of cutting conditions. Therefore, considering this random nature, fifty (50) measurements of surface roughness were taken at random locations, on each of the 27 turned surfaces. A Talysurf-4 stylus tracing surface instrument with 2.5 micrometer stylus tip width was used for obtaining the analog signals of the surface profiles. These signals were recorded on a Bruel & Kjaer 7005, FM tape recorder for further processing. A schematic diagram of the Talysurf-4 with the FM recorder is shown in Fig. 4.2. A total profile length of 10.0 mm for each signal was recorded and a stylus traversing speed of 91.4 mm/min was used for recording the profiles.

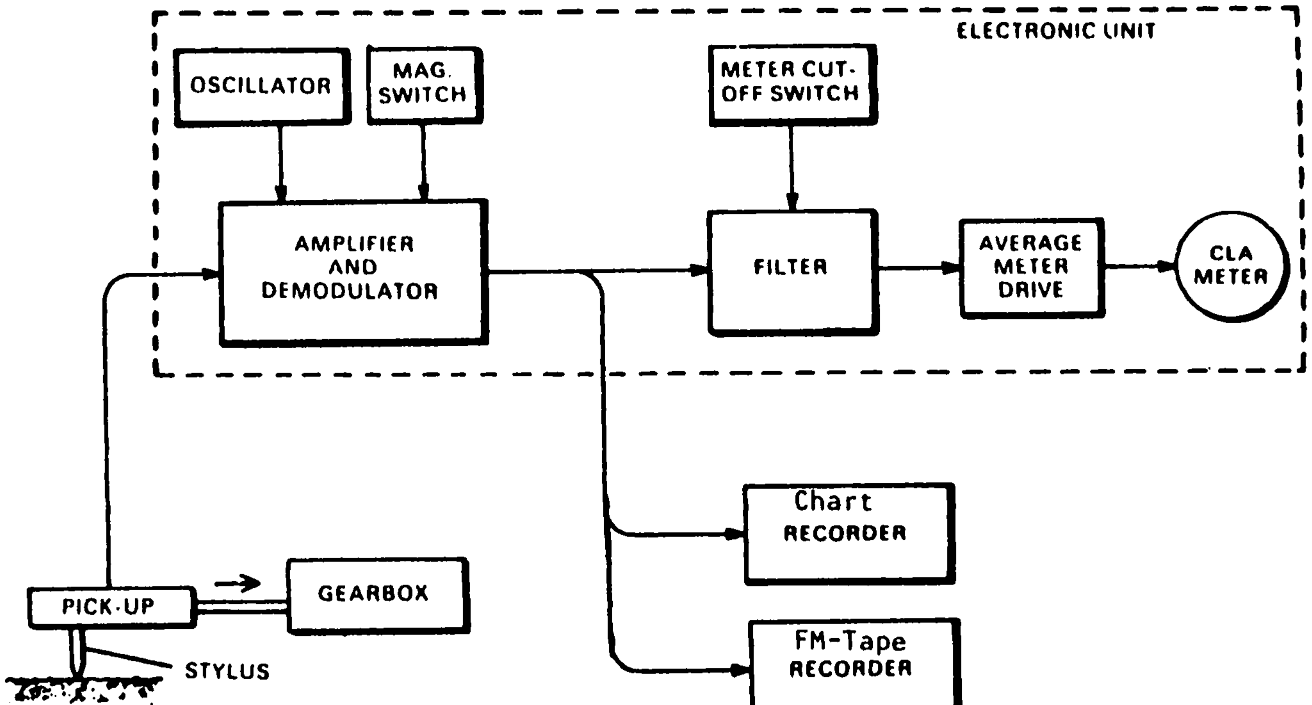


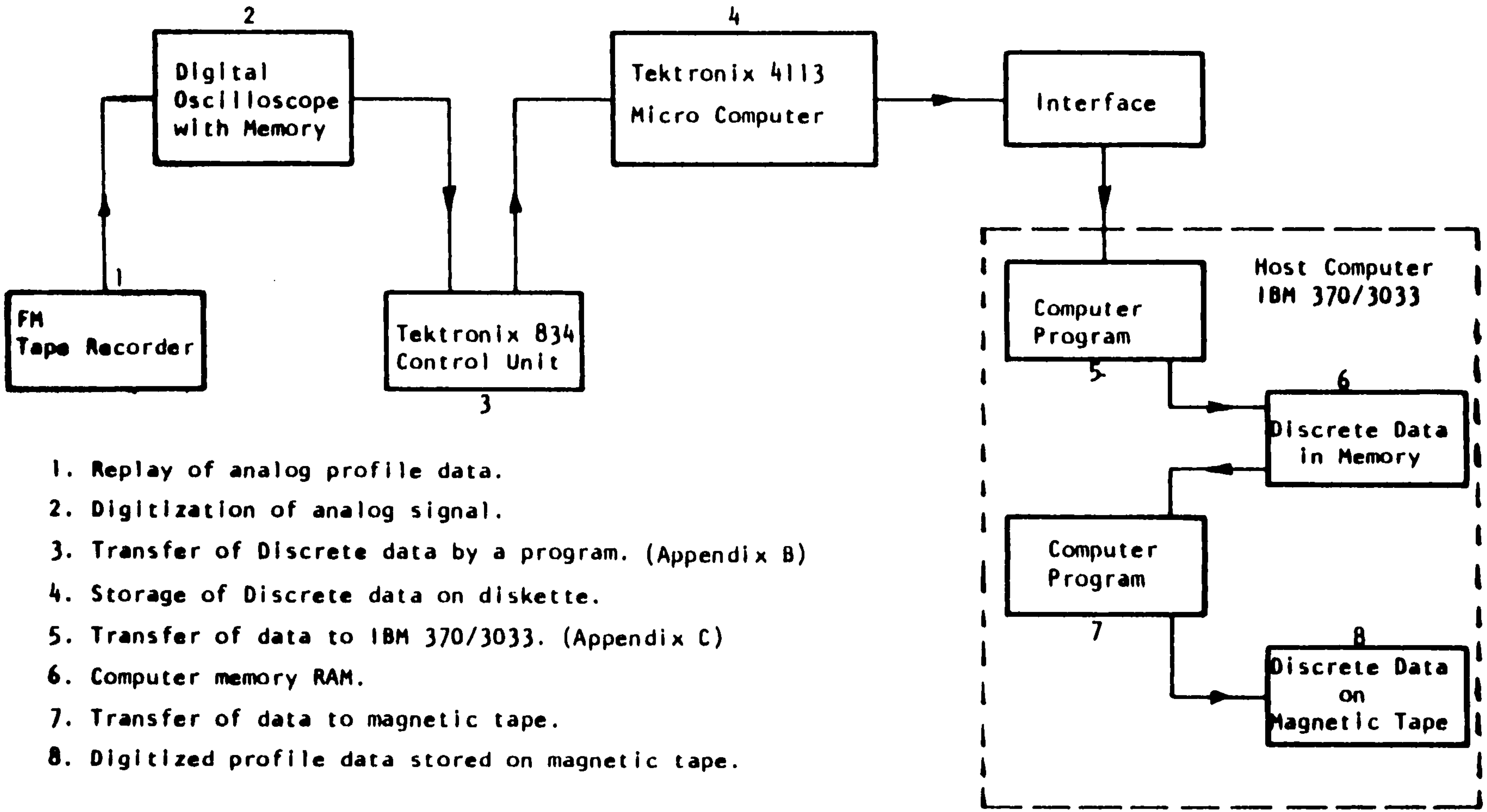
Figure 4.2 Schematic arrangement of the Talysurf-4 Surface Profile Measuring Instrument.

4.3 PROFILE SIGNAL DIGITIZATION AND DATA ACQUISITION

4.3.1 The Data Acquisition System

Since discrete data are used in the modeling procedure, the analog surface profiles data have to be digitized. A schematic diagram of the instrumentations used for profile data generation, digitization, and transmission to the host computer is shown in Fig. 4.3. The analog signal of each surface profile generated by the Talysurf-4 and recorded on a Bruel & Kjear 4 channel FM tape recorder (Section 4.2) was replayed and transferred to a Nicolet 2090 digital oscilloscope. This oscilloscope digitized the profile at a prescribed sampling interval (determined by a procedure to be described in section 4.3.2) and stored the digitized data in its memory. A Tektronix programable data communication tester, transferred the digitized profile heights from the digital oscilloscope to a Tektronix 4113 microcomputer to be recorded on the latter's $8\frac{1}{4}$ inch floppy diskette.

A computer program was written, to transfer the data recorded on the Tektronix 4113 microcomputer diskette, to the IBM 370/3033 host computer. The digitized profiles data were then recorded on a magnetic tape through a computer program for the subsequent surface roughness analysis in Chapter 5.



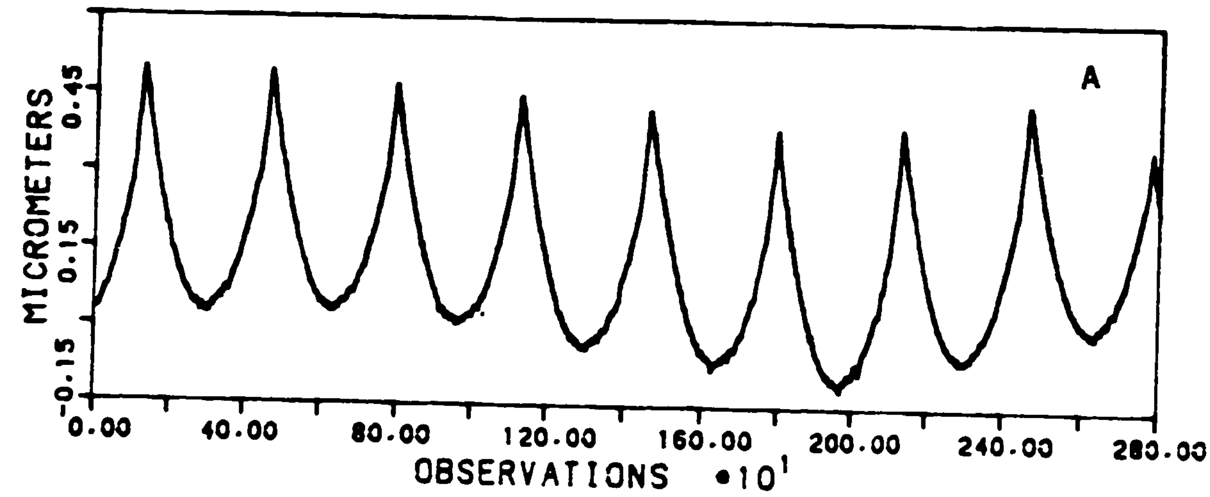
1. Replay of analog profile data.
2. Digitization of analog signal.
3. Transfer of Discrete data by a program. (Appendix B)
4. Storage of Discrete data on diskette.
5. Transfer of data to IBM 370/3033. (Appendix C)
6. Computer memory RAM.
7. Transfer of data to magnetic tape.
8. Digitized profile data stored on magnetic tape.

Figure 4.3 Schematic diagram of the Data Acquisition system

4.3.2 Optimum Sampling Interval for Digitizing Surface Profiles

The process of computing the profile characteristic parameters using digital computers requires that the profile height data be in the form of a discrete and equi-spaced set of observations. In order to have a true representation of the surface profile, the sampling interval, Δ, should be small enough to caputre significant amounts of the roughness features [35].

To find the influence of Δ on the different characteristic parameters of the surface profile, the turned surface profiles were digitized at various sampling intervals. The profile characteristic parameters were computed at each value of Δ using the computer evaluation of surface topography (CEST) [35], package to determine the optimum Δ, to be used in the surface profile digitization. Surface profiles labeled A and B in Fig. 4.4, at high and low levels of feed rate respectively, were chosen for the study. These profiles were digitized at Δ = 3 micrometer (μm). The choice of 3μm for Δ has been determined by the fact that the resolution in the data is limited to the width of Talysurf stylus tip, which is equal to 2.5 μm. The plots of Fig. 4.4 consist of 3000 profile height observations, digitized at the 3 micrometers interval.



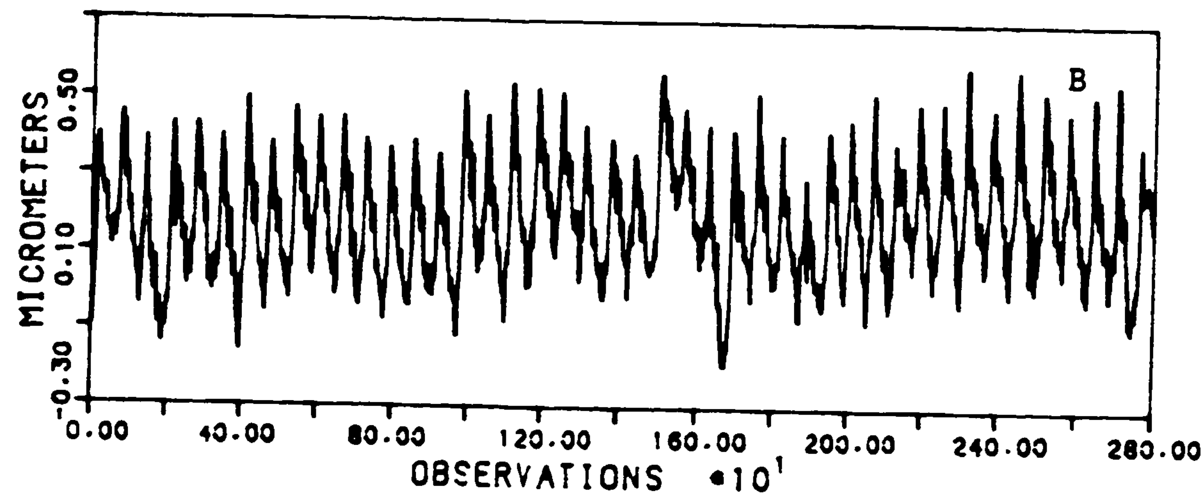


Figure 4.4 Reproduction of the turned surface profiles

A) at high level of feed rate (Test No. 1)

B) at low level of feed rate (Test No. 5)

As can be seen in Chapter 5, surface profile characteristics are commonly defined by a number of parameters; namely Center-line average R_a, Peak-to-valley height R_t, Root Mean square R_q, skewness g_1, Kurtosis g_2 and the profile spectral moments, variance of profile heights m_0, variance of slopes m_2 and variance of curvatures m_4. These characteristic parameters were computed for both profiles, at different sampling intervals (3,6,9 and 12 μm) by using CEST software package [35]. The equations used to calculate the surface profile characteristic parameters from a given set of profile heights $h_1, h_2, \ldots, h_N$ are:

$$R_a = \frac{1}{N} \sum_{i=0}^{N} |h_i|$$

$$R_q = \sqrt{\frac{1}{N} \sum_{i=0}^{N} h_i^2}$$

$$R_t = \text{Maximum profile height - Minimum profile height}$$

$$g_1 = \frac{1}{N\sigma^3} \sum_{i=1}^{N} (h_i)^3$$

$$g_2 = \frac{1}{N\sigma^4} \sum_{i=1}^{N} (h_i)^4$$

$$m_n = \int_{-\infty}^{\infty} G(\omega)\, \omega^n \, d\omega \qquad \text{for} \quad n = 0,2,4$$

where h_i's are the profile height, N is the number of profile height data points, σ^2 is the variance of heights, ω is the natural frequency and $G(\omega)$ denotes the power spectral density function defined by:

$$G(\omega) = \left(\frac{2}{\pi} \right) \int_0^{\infty} R(t) \cos(\omega t) dt$$

The variation of surface profile characteristic parameters with Δ, for profile A and B are presented in Table 4.2 respectively. The R_a, R_q, g_1 and g_2 have demonstrated insignificant variation in their values with respect to the variation in Δ whereas the spectral moments were observed to show significant variation with Δ. The variation in m_0 with increasing Δ were found to be slower than the variation in m_2 and m_4. This behavior is presented in Fig. 4.5 and Fig. 4.6 for profile A and B respectively. The ordinate corresponding to each spectral moment is normalized with respect to its value at $\Delta = 3\mu m$. The effect of Δ on the spectral moments for both the profiles A and B were found to be of the same nature.

When Fig. 4.5 and Fig. 4.6 are re-examined, it is found that m_2 and m_4 are very sensitive to the selection of sampling interval. The values of these parameters, increased significantly with a decrease in Δ, with no observed asymptote at $3\mu m$. Therefore, the smallest sampling interval which reveals that the short wave length variation was $3\mu m$. This limit was of course set by the physical dimension of the $2.5\ \mu m$ stylus width.

TABLE 4.2 Variation of Profile Characteristics With Sampling Interval

Sample	Characteristics	3 μm	6 μm	9 μm	12 μm
	R_a (μm)	10.894	10.888	10.889	10.899
	R_q (μm)	13.304	13.296	13.307	13.310
	R_t (μm)	62.431	62.159	62.059	62.301
	g_1	- 0.555	- 0.556	- 0.558	- 0.555
A	g_2	- 0.369	- 0.366	- 0.364	- 0.360
	m_0 $(\mu m)^2$	0.893	0.913	0.935	0.945
	$m_2 * 10^2$	3.321	2.939	2.276	1.583
	$m_4 * 10^6 (1/\mu m)^2$	3.461	1.707	0.855	0.432
	R_a (μm)	1.126	1.097	1.140	1.089
	R_q (μm)	1.349	1.323	1.366	1.312
	R_t (μm)	7.144	6.996	6.987	7.031
	g_1	- 0.387	- 0.406	- 0.408	- 0.397
B	g_2	- 0.585	- 0.596	- 0.585	- 0.588
	m_0 $(\mu m)^2$	0.965	0.980	1.001	1.035
	$m_2 * 10^2$	3.272	2.792	1.532	1.102
	$m_4 * 10^6 (1/\mu m)^2$	5.403	1.991	0.733	0.344

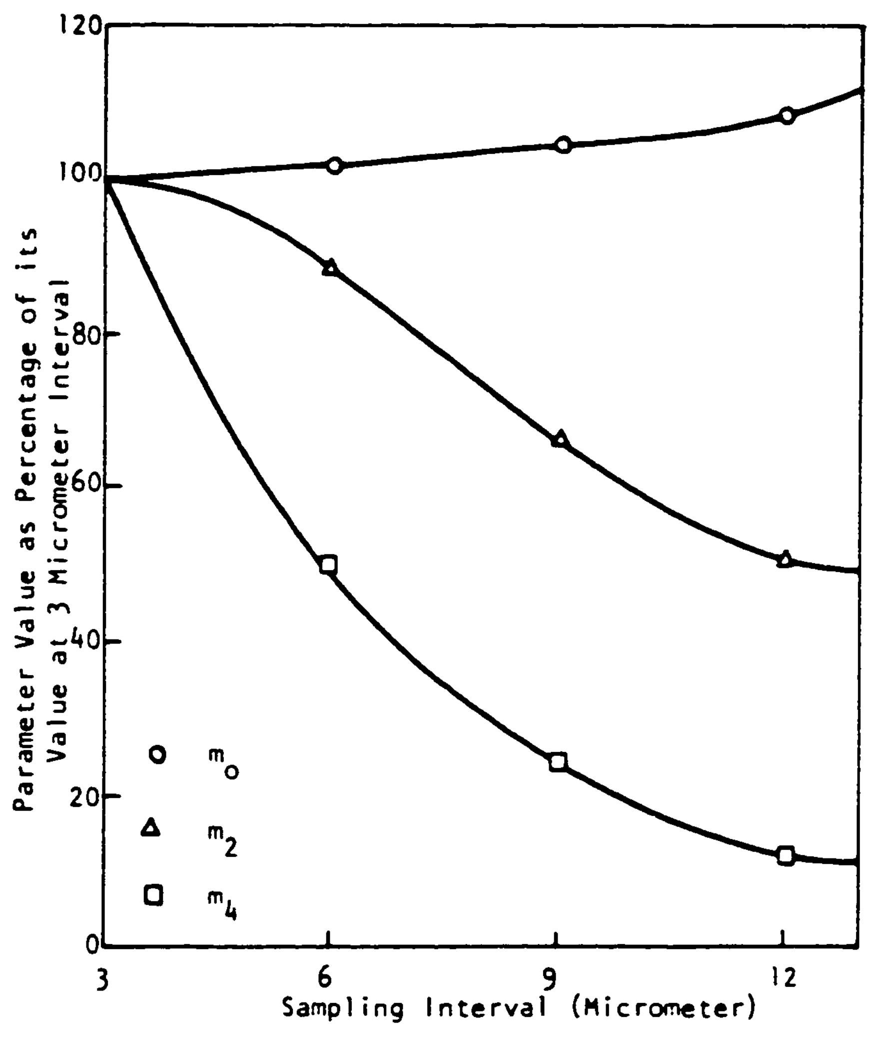


Figure 4.5 Effect of Sampling Interval on Spectral Moments of Profile A.

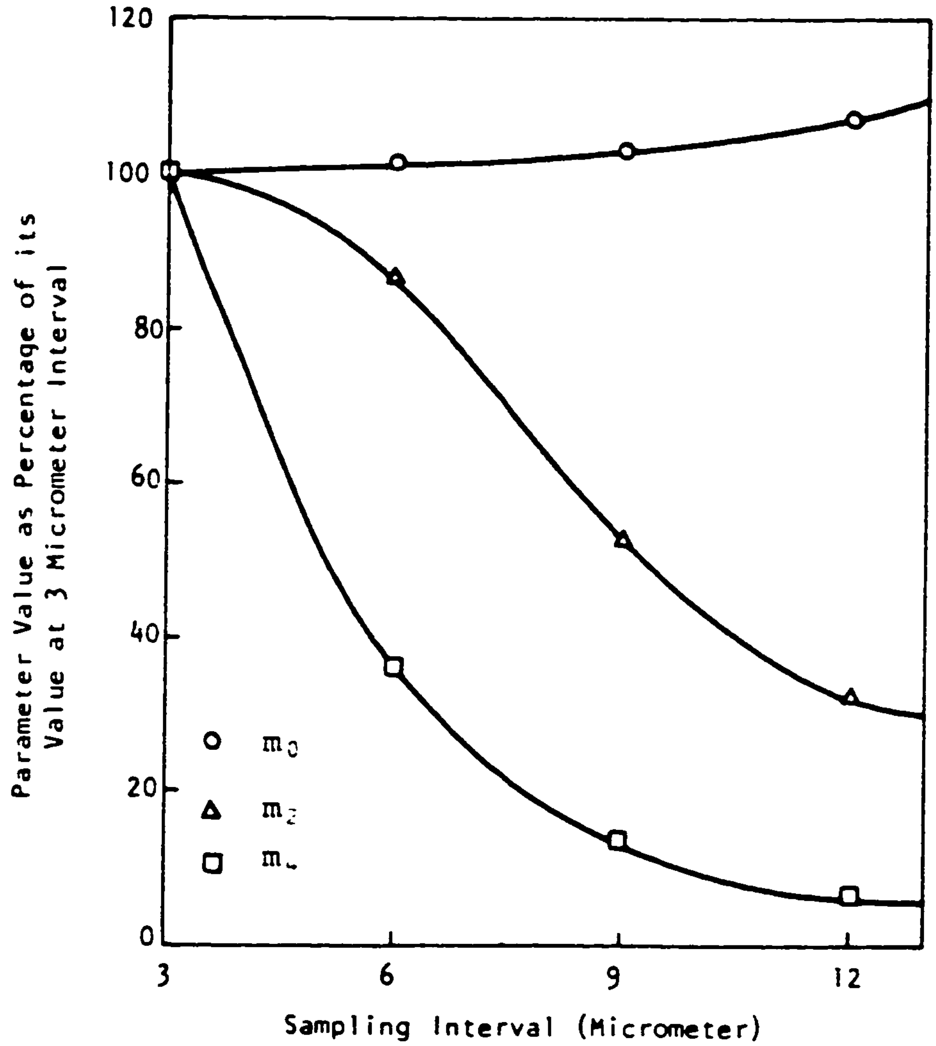


Fig. 4.6 Effect of Sampling Interval on Spectral Moments of Profile B.

5. DEVELOPMENT OF THE FINE TURNING SURFACE ROUGHNESS MODELS

Metal cutting industries use empirical or mathematical models for the adequate characterization and evaluation of various process responses, such as cutting force, surface roughness, tool life, etc. The relationship between the response and the operating cutting conditions are often developed from field or experimental data. As discussed in Cahpter 1, since the conventional modeling technique estimates the mean values of the model paramers, the response estimated from such models do not show the variability which characterizes the machining processes. In order to have better results in machining optimization and adaptive control, the models which consider the variability of the machining processes have to be developed for various process variables.

Out of the numerous machining process variables, surface roughness has found a wide consideration [5-13]. This is explained by the fact that surface roughness, being an output of the machining process, is expected to reflect the dynamics of the process, and is consequently expected to be an adequate response for the process monitoring and control.

The objective of this chapter is the application of the proposed statistical modeling technique of Chapter 3, to develop models for a number of fine turning surface roughness geometry characteristic parameters from the experimental data of Chapter 4. The developed models are to express the relationship between the random surface profile geometry characteristics and the process variables of cutting speed, feed rate and depth of cut. The parameters of the models are assumed to be random variables thus reflecting the inherent variability of the machining processes.

Section 5.1 gives an overview of the parameters used to characterize the isotropic surface roughness and the surface roughness parameters for which the new models will be developed. The sequence of steps used to develop the models using the statistical modeling technique of Chapter 3 and the fine turning experimental data of Chapter 4 are presented in Section 5.2. The procedure includes the identification of the distribution of each model parameter and the subsequent estimation of the parameters of the distributions. In Section 5.3, the statistically adequate distributions of each surface profile feature R_a, m_0, m_2 and m_4 and of their logarithms are identified from the experimental data. The obtained distribution of R_a is compared with the distributions reported in the literature. The distributions of m_0, m_2 and m_4 are given for the first time in the present work. The proposed statistical modeling technique is compared with the conventional (least square) parameter estimation technique in Section 5.4. It is concluded that the proposed technique has the advantage of fully describing the variability of the process responses. Finally, in Section 5.5 the main effects of speed, feed and depth of cut and their interactions on surface profile features R_a, m_0, m_2 and m_4 are evaluated.

5.1 CHARACTERIZATION OF SURFACE ROUGHNESS

Out of the numerous techniques developed for the characterization of surface roughness, the most common approach has been surface topography evaluation from the characteristics of a profile measured by a stylus instrument known as a profilometer. The recognized surface assessment features include center-line-average R_a, peak-to-valley height R_t, root-mean-square R_q, leveling depth and the mean depth. A surface profile can also be described by its autocorrelation function, its frequency distribution and its spectral density function. In practice, the amplitude density function is quantified by its central moments; mean, variance, skewness, g_1 and kurtosis, g_2. Similar to the amplitude density function, the power spectral density function of the surface profile height is quantified by its zeroth, second and fourth spectral moments, m_0, m_2 and m_4. These moments are used to characterize the surface profiles because they represent the variance of heights, variance of slopes and variance of curvature of the surface profile respectively [33,35]. Several other parameters have been used to characterize surface profiles. These parameters include spectrum width , mean number of zero crossing (MNZC), mean number of maxima (MNMA), root mean square crest excursion (RMCE) mean excusion spacing (MES), etc.

The empirical models which will be developed in the subsequent sections to characterize surface finish produced by fine turning will employ the features of the profile measured in the tool feeding direction. The profile features for which random models will be developed are the center line average (R_a), variance of heights (m_0), variance of slopes (m_2) and variance of curvatures (m_4).

5.2 RANDOM FINE TURNING SURFACE GEOMETRY MODELS

The functional relationship between the surface profile characteristics (R_a, m , m and m) and the independent machining variables (cutting speed, feed rate and depth of cut) is postulated in 5-13 as;

$$R = K\ V^{\alpha} F^{\beta} D^{\gamma} \tag{5.1}$$

where

R = A surface profile characteristic paramter, e.g., R_a, m_0, m_2, m_4 etc.

V = Cutting speed

F = Feed rate

D = Depth of cut

K, α, β and γ are the parameters of the postulated model, and are assumed to be random variables.

Taking the natural logarithm of both sides of Equation 5.1;

$$\ell nR = \ell nK + \alpha \ell nV + \beta \ell nF + \gamma \ell nD$$

which can be written as;

$$Y = C + \alpha X_1 + \beta X_2 + \gamma X_3 \tag{5.2}$$

where

$$\begin{aligned} Y &= \ell nR \\ C &= \ell nK \\ X_1 &= \ell nV \\ X_2 &= \ell nF \\ X_3 &= \ell nD \end{aligned} \tag{5.3}$$

Equation 5.2 is the linearized model to be fitted to the surface roughness features R_a, m_0, m_2 and m_4, where C, α, β and γ are the parameters to be obtained for each feature. The surface profile parameters, R_a, m_0, m_2 and m_4 were computed for each of the 27 test conditions from the digitized fine turning surface profile data generated in Chapter 4 using the Computer Evaluation of Surface Topography (CEST) software package [35]

Fifty values of each profile characteristic parameter (R_a, m_0, m_2 and m_4) were computed at each of the 27 test conditions of the experimental design matrix presented in Table 4.1. It should be noted here that during the development of the models the units of feed rate and depth of cut were in micrometer per revolution and micrometers respectively, while the unit of speed was in meter/min. In order to avoid an ill-conditioned matrix on using the least square technique for estimating their model parameters, the computed values of R_a and m_0 were multiplied by 100.

5.2.1 The Distributions of the Linearized Response Model Parameters

Applying the procedure of the statistical modelling technique (Chapter 3) the data sets corresponding to each surface profile feature e.g., R_a were used to compute the first 8 moments of each parameter of the linearized response model represented by Equation 5.2. The computed moments of each model parameter were then used to identify completely the probability distribution which closely describes the model parameter. The same procedure was followed for the models of the remaining profile characteristic parameters m_0, m_2 and m_4, represented by the linearized model of Equation 5.2. The set of moments computed for the parameters of the surface profile features R_a, m_0, m_2 and m_4 are listed in Table 5.1, 5.2, 5.3 and 5.4 respectively.

A) The Center-line-average, R_a, Model;

The comparison between the theoretical and computed moment ratios of the seven candidate distributions for the R_a model parameters listed in Table 5.5 to 5.8, leads to the conclusion that, the normal distribution is the adequate distribution to represent every parameter of R_a model. This is because the minimum percent deviation was attained by the normal distribution for each of parameters C, α, β and γ, of the linearized R_a model.

B) Variance of Heights, m_0, Model;

Applying the criterion for identifying the adequate distributions for m_0 model parameters C, α, β and γ Table 5.9 to 5.12 show that normal distribution has the minimum percent deviation in the corresponding set of moment ratios for each model parameter. Therefore the normal distribution is selected to be the adequate distribution for each parameter of m_0 model.

TABLE 5.1 The Computed Moments of R_a Model Random Parameters

Random Variable	Moments	r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
C	μ'_r	0.132×10^{1}	0.191×10^{1}	0.298×10^{1}	0.498×10^{1}	0.882×10^{1}	0.165×10^{2}	0.327×10^{2}	0.677×10^{2}
	μ_r	0.000×10^{0}	0.160×10^{1}	0.25×10^{-1}	0.81×10^{-1}	0.51×10^{-1}	0.66×10^{-1}	0.91×10^{-1}	0.75×10^{-1}
α	μ'_r	-0.587×10^{0}	0.379×10^{0}	-0.274×10^{0}	0.220×10^{0}	-0.196×10^{0}	0.195×10^{0}	-0.213×10^{0}	0.253×10^{0}
	μ_r	0.000×10^{0}	0.35×10^{-1}	-0.11×10^{-1}	0.37×10^{-2}	-0.793×10^{-2}	0.67×10^{-3}	-0.61×10^{-3}	0.17×10^{-3}
β	μ'_r	0.118×10^{1}	0.156×10^{1}	0.228×10^{1}	0.366×10^{1}	0.635×10^{1}	0.118×10^{2}	0.230×10^{2}	0.472×10^{2}
	μ_r	0.000×10^{0}	0.167×10^{0}	0.51×10^{-1}	0.89×10^{-1}	0.83×10^{-1}	0.76×10^{-1}	0.14×10^{-1}	0.92×10^{-1}
γ	μ'_r	0.164×10^{0}	0.49×10^{-1}	0.25×10^{-1}	0.12×10^{-1}	0.10×10^{-1}	0.71×10^{-2}	0.13×10^{-3}	0.14×10^{-3}
	μ_r	0.000×10^{0}	0.23×10^{-1}	0.91×10^{-2}	0.15×10^{-2}	0.53×10^{-2}	0.17×10^{-3}	0.90×10^{-3}	0.25×10^{-4}

μ_r = rth moment about mean μ'_r = rth momemt about origin

TABLE 5.2 The Computed Moments of m_0 Model Random Parameters

Random Variable	Moments	r=1	r=2	r=3	r=4	r=5	r=6	r =7	r=8
C	μ'_r	0.391×10^{1}	0.168×10^{2}	0.774×10^{2}	0.378×10^{3}	0.195×10^{4}	0.105×10^{5}	0.589×10^{5}	0.343×10^{6}
	μ_r	0.000×10^{0}	0.147×10^{1}	0.21×10^{-1}	0.697×10^{1}	0.79×10^{-1}	0.523×10^{2}	0.195×10^{0}	0.554×10^{3}
α	μ'_r	0.125×10^{0}	0.19×10^{-2}	0.76×10^{-2}	0.28×10^{-2}	0.58×10^{-2}	0.39×10^{-2}	0.10×10^{-2}	0.89×10^{-2}
	μ_r	0.000×10^{0}	0.32×10^{-2}	0.44×10^{-2}	0.32×10^{-4}	0.51×10^{-2}	0.52×10^{-6}	0.83×10^{-2}	0.12×10^{-7}
β	μ'_r	-0.17×10^{-1}	0.56×10^{-3}	-0.76×10^{-4}	0.47×10^{-4}	-0.91×10^{-4}	0.96×10^{-5}	-0.49×10^{-6}	0.70×10^{-7}
	μ_r	0.000×10^{0}	0.25×10^{-3}	-0.56×10^{-4}	0.20×10^{-6}	-0.91×10^{-4}	0.23×10^{-9}	-0.49×10^{-3}	0.41×10^{-12}
γ	μ'_r	-0.73×10^{-2}	0.62×10^{-4}	-0.15×10^{-4}	0.42×10^{-6}	-0.37×10^{-6}	0.16×10^{-6}	-0.67×10^{-8}	0.39×10^{-6}
	μ_r	0.000×10^{0}	0.90×10^{-5}	-0.14×10^{-5}	0.23×10^{-9}	-0.37×10^{-5}	0.10×10^{-13}	-0.66×10^{-8}	0.62×10^{-18}

μ_r = rth moment about mean μ'_r = rth momemt about origin

TABLE 5.3 The Computed Moments of m_2 Model Random Parameters

Random Variable	Moments	r=1	r=2	r=3	r=4	r=5	r=6	r =7	r=8
C	μ_r'	$0.802\text{x}10^{1}$	$0.649\text{x}10^{2}$	$0.531\text{x}10^{3}$	$0.439\text{x}10^{4}$	$0.367\text{x}10^{5}$	$0.311\text{x}10^{6}$	$0.264\text{x}10^{7}$	$0.225\text{x}10^{8}$
	μ_r	$0.000\text{x}10^{0}$	$0.683\text{x}10^{0}$	$-0.833\text{x}10^{0}$	$0.247\text{x}10^{2}$	$-0.431\text{x}10^{3}$	$0.803\text{x}10^{4}$	$-0.160\text{x}10^{6}$	$0.299\text{x}10^{7}$
α	μ_r'	$-0.243\text{x}10^{0}$	$0.67\text{x}10^{-1}$	$-0.22\text{x}10^{-1}$	$0.77\text{x}10^{-2}$	$-0.48\text{x}10^{-2}$	$0.39\text{x}10^{-2}$	$-0.83\text{x}10^{-2}$	$0.12\text{x}10^{-2}$
	μ_r	$0.000\text{x}10^{0}$	$0.87\text{x}10^{-2}$	$-0.99\text{x}10^{-3}$	$0.22\text{x}10^{-3}$	$-0.19\text{x}10^{-2}$	$0.93\text{x}10^{-3}$	$-0.55\text{x}10^{-2}$	$0.56\text{x}10^{-3}$
β	μ_r'	$0.467\text{x}10^{0}$	$0.232\text{x}10^{0}$	$0.121\text{x}10^{0}$	$0.66\text{X}10^{-1}$	$0.38\text{x}10^{-1}$	$0.22\text{x}10^{-1}$	$0.13\text{x}10^{-1}$	$0.87\text{x}10^{-2}$
	μ_r	$0.000\text{x}10^{0}$	$0.14\text{x}10^{-1}$	$0.52\text{x}10^{-3}$	$0.39\text{x}10^{-3}$	$0.44\text{x}10^{-4}$	$0.60\text{x}10^{-4}$	$0.87\text{x}10^{-4}$	$0.16\text{x}10^{-4}$
γ	μ_r'	$-0.165\text{x}10^{0}$	$0.29\text{x}10^{-1}$	$-0.14\text{x}10^{-1}$	$0.67\text{x}10^{-2}$	$-0.24\text{x}10^{-1}$	$0.22\text{x}10^{-1}$	$-0.10\text{x}10^{-1}$	$0.12\text{x}10^{-1}$
	μ_r	$0.000\text{x}10^{0}$	$0.26\text{x}10^{-2}$	$-0.18\text{x}10^{-2}$	$0.22\text{x}10^{-4}$	$-0.20\text{x}10^{-1}$	$0.29\text{x}10^{-6}$	$-0.88\text{x}10^{-1}$	$0.52\text{x}10^{-8}$

μ_r = rth moment about mean μ_r' = rth momemt about origin

TABLE 5.4 The Computed Moments of m_4 Model Random Parameters

Random Variable	Moments	r=1	r=2	r=3	r=4	r=5	r=6	r =7	r=8
C	μ'_r	0.166×10^{2}	0.285×10^{3}	0.505×10^{4}	0.920×10^{5}	0.171×10^{7}	0.329×10^{8}	0.645×10^{9}	0.129×10^{11}
	μ_r	0.000×10^{0}	0.937×10^{1}	0.24×10^{-1}	0.280×10^{3}	0.61×10^{-1}	0.134×10^{5}	0.109×10^{-1}	0.905×10^{6}
α	μ'_r	0.168×10^{0}	0.53×10^{-1}	0.35×10^{-1}	0.18×10^{-1}	0.29×10^{-1}	0.24×10^{-1}	0.54×10^{-1}	0.61×10^{-1}
	μ_r	0.000×10^{0}	0.25×10^{-1}	0.18×10^{-1}	0.17×10^{-2}	0.21×10^{-2}	0.21×10^{-3}	0.41×10^{-3}	0.36×10^{-4}
β	μ'_r	-0.182×10^{0}	0.35×10^{-1}	-0.28×10^{-1}	0.17×10^{-1}	-0.39×10^{-1}	0.37×10^{-1}	-0.86×10^{-1}	0.11×10^{-1}
	μ_r	0.000×10^{0}	0.22×10^{-2}	-0.21×10^{-2}	0.16×10^{-4}	-0.32×10^{-4}	0.18×10^{-6}	-0.63×10^{-6}	0.30×10^{-8}
γ	μ'_r	-0.129×10^{0}	0.31×10^{-1}	-0.27×10^{-1}	0.13×10^{-1}	-0.36×10^{-1}	0.26×10^{-1}	-0.71×10^{-1}	0.66×10^{-1}
	μ_r	0.000×10^{0}	0.15×10^{-1}	-0.19×10^{-1}	0.69×10^{-3}	-0.32×10^{-3}	0.52×10^{-4}	-0.59×10^{-4}	0.53×10^{-5}

μ_r = rth moment about mean μ'_r = rth momemt about origin

TABLE 5.5 *The Theoretical and Computed Moment Ratios of Parameter C of the R_α Model*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.15	16.16	115.00				
		c	0.0	5.02	7.74	9.64				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
	$/\ln(\mu_2'/\mu_1'^2)$	b	0.00	1.00	2.86	5.50	8.83	12.82	17.40	22.52
		c	0.00	0.00	4.44	8.33	11.64	14.52	17.12	19.56
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.15	16.16	115.02				
		c	0.00	75.04	319.77	1177.98				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.09	1.28	1.60	2.16	3.06	4.56	7.12
		c	0.00	45.46	78.60	93.27	98.19	99.57	99.91	99.98
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.12	0.23	0.34	0.45	0.55	0.65	0.74
		c	0.00	93.96	94.15	94.30	94.40	94.50	94.57	94.67
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$	a	0.00	1.00	2.00	3.00	8.00	5.00	6.00	7.00
	$/\mu_{r-1}'(\mu_2' - \mu_1'^2)$	b	0.00	1.00	1.80	2.65	3.41	4.06	4.83	5.17
		c	0.00	0.00	10.00	11.66	14.75	18.80	19.50	26.14
Beta	$(\mu_2' - \mu_1')(\mu_r'/\mu_{r-1}' - \mu_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.54	1.80	2.14	2.32	2.46	2.57
	$/(\mu_1'^2 - \mu_2')(1 - \mu_r'/\mu_{r-1}')$	c	0.00	0.00	22.67	36.88	46.53	53.55	58.95	63.26

a= Theoretical b= Computed c= % Deviation

TABLE 5.6 *The Theoretical and Computed Moment Ratios of Parameter α of the R_a Model*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.10	15.92	113.76				
		c	0.0	3.31	6.16	8.34				
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	3.16	6.39	10.70	16.39	23.65	31.96
	$/\ln(\mu'_2/\mu_1'^2)$	c	0.00	0.00	5.52	6.54	7.50	9.27	12.62	14.14
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.10	15.92	113.76				
		c	0.00	72.18	313.62	1164.03				
Exponential	$\mu'_r/(\mu'_1)^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.10	1.35	1.85	2.82	4.76	8.86	17.92
		c	0.00	44.95	77.39	92.28	97.65	99.34	99.82	99.96
Chi-square	$\mu'_r/\mu'_{r-1} - \mu'_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	-0.06	-0.14	-0.21	-0.31	-0.41	-0.51	-0.60
		c	0.00	102.97	103.41	103.57	103.83	104.05	104.22	104.28
Gamma	$\mu'_r\mu'_1 - \mu_1'^2\mu'_{r-1}$	a	0.00	1.00	2.00	3.00	8.00	5.00	6.00	7.00
		b	0.00	1.00	2.29	3.60	5.15	6.82	8.53	10.11
	$/\mu'_{r-1}(\mu'_2 - \mu_1'^2)$	c	0.00	0.00	14.69	20.18	28.83	36.42	42.16	44.43
Beta	$(\mu'_2 - \mu'_1)(\mu'_r/\mu'_{r-1} - \mu_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	2.19	3.29	4.48	5.64	6.75	8.31
	$/(\mu_1'^2 - \mu'_2)(1 - \mu'_r/\mu_{r-1})$	c	0.00	0.00	9.58	9.86	12.05	12.75	12.50	18.71

a= Theoretical b= Computed c= % Deviation

TABLE 5.7 *The Theoretical and Computed Moment Ratios of Parameter β of the R_a Model*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.19	16.20	117.21				
		c	0.0	6.62	8.01	11.63				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$ $/\ln(\mu_2'/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.91	5.61	9.00	12.97	17.43	22.28
		c	0.00	0.00	2.96	6.42	9.95	13.49	17.00	20.42
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.19	16.20	117.21				
		c	0.00	77.69	320.83	1202.36				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.12	1.39	1.89	2.78	4.36	7.24	12.55
		c	0.00	43.98	76.80	92.12	97.68	99.39	99.85	99.95
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.14	0.28	0.42	0.55	0.67	0.77	0.87
		c	0.00	92.90	92.85	92.93	93.07	93.28	93.53	93.80
Gamma	$\mu_r'\mu_1'-\mu_1'^2\mu_{r-1}'$ $/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	a	0.00	1.00	2.00	3.00	8.00	5.00	6.00	7.00
		b	0.00	1.00	2.02	2.98	3.19	4.74	5.47	6.01
		c	0.00	0.00	0.78	0.40	2.36	5.24	8.79	14.14
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1)$ $/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.39	1.59	1.71	1.78	1.84	1.88
		c	0.00	0.00	30.39	46.92	57.19	64.21	69.31	73.16

a= Theoretical b= Computed c= % Deviation

TABLE 5.8 The Theoretical and Computed Moment Ratios of Parameter γ of the R_a Model

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	2.96	14.30	96.93				
		c	0.0	1.46	4.58	7.68				
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$ $/\ln(\mu'_2/\mu'^2_1)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.82	4.58	7.29	9.67	13.61	16.66
		c	0.00	0.00	5.98	23.54	27.06	35.50	35.20	40.47
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	2.96	14.30	96.93				
		c	0.00	64.24	271.74	977.06				
Exponential	$\mu'_r/(\mu'_1)^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.84	5.56	16.30	84.56	360.00	3939.90	25349.12
		c	0.00	8.12	7.29	32.09	29.53	49.99	21.82	37.13
Chi-square	$\mu'_r/\mu'_{r-1} - \mu'_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.14	0.33	0.31	0.69	0.54	1.63	0.89
		c	0.00	93.11	91.67	94.71	91.39	94.64	86.37	93.62
Gamma	$\mu'_r\mu'_1 - \mu'^2_1\mu'_{r-1}$ $/\mu'_{r-1}(\mu'_2 - \mu'^2_1)$	a	0.00	1.00	2.00	3.00	8.00	5.00	6.00	7.00
		b	0.00	1.00	2.40	2.30	5.00	3.78	9.87	12.02
		c	0.00	0.00	21.00	23.18	25.04	22.21	64.50	71.17
Beta	$(\mu'_2 - \mu_1)(\mu_r/\mu'_{r-1} - \mu'_1)$ $/(\mu'^2_1 - \mu'_2)(1 - \mu'_r/\mu'_{r-1})$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	3.36	3.10	23.74	9.04	-10.37	-78.65
		c	0.00	0.00	68.09	3.43	493.47	80.85	272.86	1223.60

a= Theoretical b= Computed c= % Deviation

TABLE 5.9 *The Theoretical and Computed Moment Ratios of Parameter C of the m_0 Model*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.19	16.21	116.07				
		c	0.00	6.35	8.08	10.55				
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$ $/\ln(\mu'_2/\mu'^2_1)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.76	5.17	8.14	11.62	15.53	19.85
		c	0.00	0.00	7.90	13.82	18.57	22.56	26.03	29.11
Uniform	$\mu'_{2r}/(\mu'_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.19	16.21	116.07				
		c	0.00	77.24	321.08	1189.70				
Exponential	$\mu'_r/(\mu'_1)^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.09	1.29	1.61	2.12	2.91	4.18	6.22
		c	0.00	45.18	78.51	93.29	98.24	99.59	99.92	99.98
Chi-square	$\mu'_r/\mu'_{r-1} - \mu'_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.38	0.69	0.97	1.23	1.47	1.70	1.91
		c	0.00	81.12	82.75	83.81	84.60	85.25	85.83	86.36
Gamma	$\mu'_r\mu'_1 - \mu'^2_1\mu'_{r-1}$ $/\mu'_{r-1}(\mu'_2 - \mu'^2_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.83	2.57	3.26	3.91	4.50	5.06
		c	0.00	0.00	8.63	14.23	18.41	21.86	24.92	27.72
Beta	$(\mu'_2 - \mu'_1)(\mu'_r/\mu'_{r-1} - \mu'_1)$ $/(\mu'^2_1 - \mu'_2)(1 - \mu'_r/\mu'_{r-1})$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.67	2.18	2.59	2.93	3.21	3.45
		c	0.00	0.00	16.55	27.34	35.22	41.39	46.44	50.67

a= Theoretical b= Computed c= % Deviation

TABLE 5.10 The Theoretical and Computed Moment Ratios of Parameter α of the m_0 Model

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.17	15.91	114.18				
		c	0.00	5.64	6.08	8.75				
Lognormal	$\ln(\mu'_r/(\mu'_1)^r) / \ln(\mu'_2/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	7.31	13.14	23.34	37.48	53.68	64.26
		c	0.00	0.00	143.78	119.09	133.40	149.90	155.63	129.50
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.17	15.91	114.18				
		c	0.00	76.07	313.31	1168.73				
Exponential	$\mu'_r/(\mu'_1)^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.20	3.87	11.40	190.07	1033.92	20743.50	147034.66
		c	0.00	39.83	35.44	52.48	58.39	63.39	311.58	264.67
Chi-square	$\mu'_r/\mu'_{r-1} - \mu_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.03	0.27	0.25	1.96	0.56	2.39	0.76
		c	0.00	98.73	93.05	95.94	75.47	94.44	80.09	94.55
Gamma	$\mu'_r\mu'_1-\mu_1^2\mu'_{r-1} / \mu_{r-1}(\mu_2-\mu_1^2)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	10.90	19.56	77.01	21.80	93.71	29.93
		c	0.00	0.00	445.40	552.00	1852.28	336.50	1461.89	327.56
Beta	$(\mu'_2-\mu'_1)(\mu'_r/\mu'_{r-1} - \mu'_1) / (\mu_1'^2-\mu'_2)(1-\mu'_r/\mu'_{r-1})$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	15.52	25.86	60.12	58.18	52.58	226.90
		c	0.00	0.00	676.16	762.00	1603.09	1603.67	976.45	3141.49

a= Theoretical b= Computed c= % Deviation

TABLE 5.11 *The Theoretical and Computed Moment Ratios of Parameter β of the m_0 Model*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	3.14	14.36	100.76				
		c	0.00	4.57	4.26	4.04				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$	b	0.00	1.00	4.14	9.56	18.32	21.34	34.79	38.29
	$/\ln(\mu'_2/\mu_1'^2)$	c	0.00	0.00	47.06	64.92	83.27	42.28	65.70	36.78
		a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}/(\mu_2)^r$	b	1.00	3.14	14.36	100.76				
		c	0.00	76.29	273.03	1019.55				
		a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
Exponential	$\mu'_r/(\mu'_1)^r$	b	1.00	1.81	13.73	49.13	5368.70	9135.0	20743.50	147034.66
		c	0.00	9.46	128.88	104.71	4429.39	263.39	4311.58	34264.67
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu'_r/\mu'_{r-1} - \mu'_1$	b	0.00	-0.01	-0.12	-0.05	-0.06	-0.08	-51.99	-0.12
		c	0.00	100.72	102.90	106.76	338.70	431.87	533.29	100.88
Gamma	$\mu'_r\mu'_1-\mu_1'^2\mu'_{r-1}$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	2.12	3.18	4.64	6.16	7.22	8.62
	$/\mu_{r-1}(\mu'_2-\mu_1'^2)$	c	0.00	0.00	6.00	5.95	16.00	23.11	20.11	23.21
Beta	$(\mu'_2-\mu'_1)(\mu'_r/\mu'_{r-1} - \mu_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	2.62	3.08	8.54	5.75	17.16	8.69
	$/(\mu_1'^2-\mu'_2)(1-\mu'_r/\mu'_{r-1})$	c	0.00	0.00	31.00	2.85	113.50	14.89	186.00	24.14

a= Theoretical b= Computed c= % Deviation

TABLE 5.12 The Theoretical and Computed Moment Ratios of Parameter γ of the m_0 Model

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	2.89	13.84	94.05				
		c	0.00	3.66	7.73	10.42				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$ $/\ln(\mu_2'/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	23.31	32.00	77.24	88.67	143.90	157.18
		c	0.00	0.00	677.13	433.37	672.39	491.15	585.28	461.38
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	2.89	13.84	94.05				
		c	0.00	60.56	259.51	945.04				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.17	38.49	150.04	968.70	2995.09	68743.50	287089.78
		c	0.00	41.52	541.55	555.16	667.89	987.90	5411.58	94565.32
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	-0.01	-0.23	-0.21	-8.67	-0.64	-41.56	-1.65
		c	0.00	100.06	105.81	103.51	208.56	106.36	446.33	111.78
Gamma	$\mu_r'\mu_1'-\mu_1'^2\mu_{r-1}'$ $/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	18.28	29.09	7024.13	29.45	3364.09	41.28
		c	0.00	0.00	865.90	869.66	175503.0	488.97	560568.1	489.68
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$ $/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	15.14	26.76	731.40	28.45	797.00	39.34
		c	0.00	0.00	657.00	792.00	18185.50	469.14	13183.41	461.96

a= Theoretical b= Computed c= % Deviation

C) Variance of Slopes, m_2, Model

The moment ratios presented in Table 5.13 for m_2-model parameter C shows that the lognormal distribution has the minimum percent deviation of 7.14, followed by the other distributions which exhibit significantly large deviations. The lognormal distribution is therefore selected to be an adequate distribution for C. Similarly it is concluded from Tables 5.14 and 5.16, that both α and γ are represented by normal distributions where as the moment ratios of β shown in Table 5.15 indicate that an adequate distribution to represent β is the beta distribution.

D) Variance of Curvature, m_4, Model

The moment ratios of the m_4 model parameter C are presented in Table 5.17. From this table it is observed that the percent deviation of the normal, gamma and lognormal distributions are 10.77, 13.71 and 14.38 respectively. These values are relatively close and could indicate that the parameter C may be represented by any of the three distributions. This behavior has been discussed in Section 3.6. It has been reported that if the coefficient of variation ($\frac{\sigma}{\mu}$) of any random variable is

TABLE 5.13 The Theoretical and Computed Moment Ratios of Parameter C of the m_2 Model

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.0	105.0				
		b	1.00	52.87	215130.	13700170.0				
		c	0.00	1662.3	14341.0	130476.8				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$ $/\ln(\mu_2'/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.82	5.80	9.62	14.78	20.30	26.00
		c	0.00	0.00	6.00	3.27	3.83	1.50	3.33	7.14
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	52.87	215130.	13700170.				
		c	0.00	2837.	55877.0	1522240.1				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.01	1.03	1.06	1.11	1.17	1.24	1.32
		c	0.00	49.50	82.80	95.57	99.07	99.98	99.97	99.99
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.08	0.16	0.26	0.33	0.45	0.48	0.50
		c	0.00	95.74	96.00	95.67	95.87	95.50	95.93	96.40
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$ $/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	a	0.00	1.00	2.00	3.00	8.00	5.00	6.00	7.00
		b	0.00	1.00	1.81	2.94	3.74	5.02	5.35	5.53
		c	0.00	0.00	8.50	0.50	3.25	5.50	5.77	16.43
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1)$ $/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1})$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.81	2.94	3.74	5.02	5.35	5.53
		c	0.00	0.00	9.50	1.80	6.46	0.35	10.75	21.00

a= Theoretical b= Computed c= % Deviation

TABLE 5.14 The Theoretical and Computed Moment Ratios of Parameter α of the m_2 Model

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	2.96	14.15	97.74				
		c	0.00	1.23	5.67	7.26				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$ $/\ln(\mu_2'/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	3.01	5.82	12.66	21.35	37.10	50.25
		c	0.00	0.00	0.15	3.00	26.60	42.33	76.70	79.50
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	2.96	14.15	97.74				
		c	0.00	64.45	267.53	986.00				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.15	1.51	2.23	5.74	19.05	167.08	1027.97
		c	0.00	42.50	74.80	90.70	95.20	97.35	96.68	97.50
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	-0.04	-0.08	-0.12	-0.38	-0.56	-1.88	-1.25
		c	0.00	98.20	98.07	98.08	95.25	94.37	84.30	91.11
Gamma	$\mu_r'\mu_1'-\mu_1'^2\mu_{r-1}'$ $/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	a	0.00	1.00	2.00	3.00	8.00	5.00	6.00	7.00
		b	0.00	1.00	2.15	3.21	10.61	15.67	52.50	34.81
		c	0.00	0.00	7.70	7.00	165.00	213.40	775.00	397.14
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1)$ $/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	2.08	3.02	8.36	11.09	21.46	17.86
		c	0.00	0.00	4.30	0.67	108.90	122.00	258.33	155.14

a= Theoretical b= Computed c= % Deviation

TABLE 5.15 The Theoretical and Computed Moment Ratios of Parameter β of the m_2 Model

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	2.14	23.95	9773.38				
		c	0.00	28.76	59.68	44.96				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$ $/\ln(\mu_2'/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.91	5.59	8.94	12.81	17.28	22.34
		c	0.00	0.00	3.11	6.68	10.63	14.59	17.71	20.21
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	2.14	23.95	4773.38				
		c	0.00	18.73	522.15	529.37				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.06	1.19	1.40	1.72	2.17	2.84	3.86
		c	0.00	48.88	80.13	94.16	98.57	99.69	99.94	99.99
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.03	0.06	0.08	0.10	0.12	0.14	0.17
		c	0.00	98.54	98.57	98.62	98.69	98.77	98.79	98.80
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}$ $/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	a	0.00	1.00	2.00	3.00	8.00	5.00	6.00	7.00
		b	0.00	1.00	1.96	2.83	3.58	4.23	4.98	5.75
		c	0.00	0.00	1.97	5.45	10.31	15.31	17.03	17.96
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$ $/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	2.07	3.17	4.22	5.21	6.46	7.71
		c	0.00	0.00	3.79	5.78	5.46	4.17	7.73	10.17

a= Theoretical b= Computed c= % Deviation

TABLE 5.16 The Theoretical and Computed Moment Ratios of Parameter γ of the m_2 Model

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.27	16.89	114.98				
		c	0.00	9.00	12.60	9.50				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$ $/\ln(\mu_2'/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	12.55	24.15	58.24	77.18	113.52	135.45
		c	0.00	0.00	318.70	302.50	482.40	414.53	440.57	383.75
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.27	16.89	114.98				
		c	0.00	81.67	338.70	1177.56				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.09	3.12	8.95	197.56	1102.17	29828.12	218352.4
		c	0.00	45.50	48.00	62.71	64.63	53.08	491.83	441.55
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	-0.02	-0.31	-0.31	-3.48	-0.76	-4.31	-1.05
		c	0.00	99.00	92.25	94.83	56.50	92.40	64.08	92.50
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$ $/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	a	0.00	1.00	2.00	3.00	8.00	5.00	6.00	7.00
		b	0.00	1.00	19.49	19.65	221.83	48.19	274.33	66.53
		c	0.00	0.00	875.00	555.00	5445.00	864.00	4472.00	850.00
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$ $/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	15.64	15.74	56.33	29.61	59.17	35.54
		c	0.00	0.00	682.00	425.00	1308.25	492.00	886.00	407.70

a= Theoretical b= Computed c= % Deviation

TABLE 5.17 *The Theoretical and Computed Moment Ratios of Parameter C of the m_4 Model*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.19	16.29	117.36				
		c	0.00	6.36	8.61	10.77				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
	$/\ln(\mu_2'/\mu_1'^2)$	b	0.00	1.00	3.16	5.65	9.16	13.42	18.36	23.97
		c	0.00	0.00	3.16	5.92	8.35	10.54	12.54	14.38
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.19	16.29	117.36				
		c	0.00	77.26	321.15	1203.98				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.03	1.10	1.21	1.36	1.56	1.85	2.22
		c	0.00	48.30	81.64	94.97	98.86	99.78	99.96	99.99
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.56	1.09	1.59	2.07	2.53	2.98	3.42
		c	0.00	71.81	72.72	73.48	74.11	74.65	75.13	75.57
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.90	2.81	3.65	4.44	5.26	6.04
	$/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	c	0.00	0.00	5.00	6.33	8.75	11.20	12.33	13.71
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.87	2.65	3.36	4.00	4.60	5.15
	$/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	c	0.00	0.00	6.33	11.57	16.00	19.86	23.29	26.58

a= Theoretical b= Computed c= % Deviation

less than 0.24, the normal and lognormal distributions become very similar. Also it is known that gamma distribution approaches normal distribution when the former's shape parameter α is significantly high as compared to its scale parameter [32]. From the moments of the parameter C, the coefficient of variation was found to be 0.18. The shape parameter α of gamma was found to be 29.5, while its scale parameter θ was found to be equal to 1.7. Since the computed C.O.V. = 0.18 is less than 0.24 and the gamma distribution parameter α = 29.5 is significantly large when compared with θ =1.7, it is justified that both lognormal and gamma approach the normal distribution. However, inspite of the small difference in the percent deviation, the normal distribution was selected tc be an adequate distribution for the random parameter C because normal distribution had the smallest percent deviation. As for the model parameters, α, β and γ, Tables 5.18 to 5.20 show that the normal distribution moment ratios have the minimum percent deviation for all model parameters while the other distributions show significantly large deviations. Therefore the normal distribution is an adequate distribution for the parameter α, β and γ.

A summary of the statistically adequate distributions of the parameters of the surface profile features

TABLE 5.18 The Theoretical and Computed Moment Ratios of Parameter α of the m_4 Model

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	2.91	14.10	96.98				
		c	0.00	3.07	5.98	7.63				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	b	0.00	1.00	3.18	5.03	8.57	11.14	15.28	18.31
	$/\ln(\mu_2'/\mu_1'^2)$	c	0.00	0.00	6.06	16.17	14.27	25.72	27.25	34.62
		a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}/(\mu_2)^r$	b	1.00	2.91	14.10	96.98				
		c	0.00	61.55	266.28	977.64				
		a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
Exponential	$\mu_r'/(\mu_1')^r$	b	1.00	1.87	7.32	23.27	213.63	1065.40	14172.31	94301.07
		c	0.00	6.52	22.03	3.03	78.03	47.97	181.19	133.88
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	b	0.00	0.15	0.49	0.37	1.38	0.67	2.07	0.95
		c	0.00	92.67	87.72	93.88	82.78	93.78	82.73	93.19
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	3.35	2.50	9.41	4.58	14.14	6.50
	$/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	c	0.00	0.00	67.68	116.51	135.21	8.29	135.78	9.12
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	6.75	3.69	-11.86	19.62	-7.81	-36.94
	$/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	c	0.00	0.00	237.46	23.07	395.03	292.47	230.26	627.73

a= Theoretical b= Computed c= % Deviation

TABLE 5.19 *The Theoretical and Computed Moment Ratios of Parameter β of the m_4 Model*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.19	16.35	118.35				
		c	0.00	6.36	8.99	12.68				
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$ $/\ln(\mu'_2/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	23.42	41.48	80.33	105.60	144.39	173.11
		c	0.00	0.00	680.78	591.44	703.26	604.00	587.60	518.25
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.19	16.35	118.35				
		c	0.00	77.27	324.67	1214.62				
Exponential	$\mu'_r/(\mu'_1)^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.07	4.65	15.20	194.25	194.25	1019.42	85422.67
		c	0.00	46.61	22.53	36.66	61.87	41.58	157.73	111.86
Chi-square	$\mu'_r/\mu'_{r-1} - \mu'_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	-0.01	-0.61	-0.41	-2.14	-0.77	-2.14	-1.02
		c	0.00	100.62	115.26	106.88	126.80	107.73	117.81	107.25
Gamma	$\mu'_r\mu'_1 - \mu_1'^2\mu'_{r-1}$ $/\mu'_{r-1}(\mu'_2-\mu_1'^2)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	49.46	33.48	173.73	62.66	173.19	82.25
		c	0.00	0.00	2373.01	1016.22	4243.36	1153.19	2786.60	1075.03
Beta	$(\mu'_2-\mu'_1)(\mu'_r/\mu'_{r-1} - \mu'_1)$ $/(\mu_1'^2-\mu'_2)(1-\mu'_r/\mu'_{r-1})$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	32.95	25.07	62.38	38.27	62.31	44.71
		c	0.00	0.00	1547.82	735.68	1459.57	665.46	938.56	538.73

a= Theoretical b= Computed c= % Deviation

TABLE 5.20 The Theoretical and Computed Moment Ratios of Parameter γ of the m_4 Model

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	3.17	16.43	113.11				
		c	0.00	5.77	9.58	7.72				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$	b	0.00	1.00	4.04	5.42	10.99	13.73	18.60	21.72
	$/\ln(\mu'_2/\mu_1'^2)$	c	0.00	0.00	34.76	8.00	9.89	8.50	11.51	22.45
		a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}/(\mu_2)^r$	b	1.00	3.17	16.43	113.11				
		c	0.00	76.28	326.94	1156.75				
		a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
Exponential	$\mu'_r/(\mu'_1)^r$	b	1.00	1.87	12.61	44.63	982.34	5464.69	14808.79	818072.30
		c	0.00	6.41	110.19	85.95	718.62	658.98	2177.90	1928.95
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu'_r/\mu'_{r-1}-\mu'_1$	b	0.00	-0.11	-0.75	-0.33	-2.73	-0.59	-2.60	-2.79
		c	0.00	105.66	118.63	115.49	134.11	105.93	121.65	105.68
Gamma	$\mu'_r\mu'_1-\mu_1'^2\mu'_{r-1}$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	6.58	2.91	24.11	5.23	22.95	7.03
	$/\mu'_{r-1}(\mu'_2-\mu_1'^2)$	c	0.00	0.00	229.01	2.95	502.50	4.67	282.50	0.36
Beta	$(\mu'_2-\mu'_1)(\mu'_r/\mu'_{r-1}-\mu'_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	4.36	2.48	7.76	3.76	7.65	4.54
	$/(\mu_1'^2-\mu'_2)(1-\mu'_r/\mu'_{r-1})$	c	0.00	0.00	118.14	17.34	94.09	24.46	27.53	35.20

a= Theoretical b= Computed c= % Deviation

R_a, m_0, m_2 and m_4 models is given in Table 5.21. The sets of moments corresponding to each model parameter (Tables 5.1 to 5.4) will be used to obtain the parameters of the distributions representing each model parameter, in Section 5.2.3.

5.2.2. Distributions of the Postulated Response Model Parameters

In order to simplify the development of the machining process response models, Equation 5.1 has been transformed into the linear form of Equation 5.2 and the statistical modeling technique was used to identify the distribution of the linearized model parameters C, α, β and γ. When Equation 5.1 and 5.2 are compared it is clear that the two models possess α, β and γ as common parameters, whereas the parameters C and K are related by

$$C = \ln K$$

Since the analysis of Section 5.2 results in the distribution of C, it is important to explicitly identify the distribution of K (which is a function of random parameter C) to fully define the model represented by Equation 5.1.

It has already been established in the preceding chapters that the distribution of any random variable can be identified if its moments are known. In the following

TABLE 5.21 Distributions of The Model Parameters of R_a, m_0, m_2, and m_4

Model	Accepted Distributions of the Parameters C	α	β	γ
R_a	Normal	Normal	Normal	Normal
m_0	Normal	Normal	Normal	Normal
m_2	Lognormal	Normal	Beta	Normal
m_4	Normal	Normal	Normal	Normal

two approaches for obtaining the moments of $K = \exp(C)$ will be described.

5.2.2.1 The Probability Distribution Method

In this method the moments of random parameter K will be obtained from the known probability density function of the random parameter C. Since the logarithm function is monotonically increasing, then the probability that random variable K is less than or equal to k is same as the probability that ℓnK is less than or equal to ℓnk that is

$$P_r\ (K \leq k) = P_r\ (\ell nK \leq \ell nk)$$

where P_r denotes the probability. From the definition of cumulative density function;

$$F_K(k) = F_{\ell nK}\ (\ell nk)$$

or

$$F_K(k) = F_C(t) \tag{5.4}$$

where $C = \ell nK$, $t = \ell nk$ and F denotes the cumulative density function (c.d.f) of the respective random parameters. Then

$$f_K(k)\ dk = f_C(t)\, dt \tag{5.5}$$

where f denotes the probability density function (p.d.f.) of the respective random parameters. It should be noted here that our intention is to estimate the moments of K from moments of C $(K = \exp(C))$, i.e., we have to obtain $E(K^r)$ by knowing $E(e^{Cr})$.

Consider

$$E(e^{rt}) = \int_{\ell_t}^{U_t} e^{rt} f_C(t)dt \tag{5.6}$$

where ℓ_t and U_t are the lower and upper limits of the random variable C. The quantity in Equation 5.6, could be computed for every r $(r = 1, 2, \ldots)$, since $t = \ell n\, k$ then

$$\begin{aligned}\int_{\ell_t}^{U_t} e^{rt} f_c(t)dt &= \int_{\ell_K}^{U_K} e^{r\ell nk} \quad f_k(k)dk\\ &= \int_{\ell_K}^{U_K} k^r f_k(k)dk\\ &= E(K^r)\\ &= \mu'_{rK}\end{aligned} \tag{5.7}$$

where $\ell_k = e^{\ell t}$ and $U_K = e^{U_t}$

Therefore, one could compute the rth moment of random parameter K by knowing the density function, $f_C(t)$ the random parameter C from Equation 5.7.

The following example illustrates the use of this results. Suppose C is normally distributed that is

$$f_C(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-1/2\sigma^2(t-\mu)^2}$$

where μ and σ^2 are the mean and variance of the distribution respectively.

From Equation 5.7

$$\mu'_{rK} = \int_{-\infty}^{\infty} e^{rt} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-1/2\sigma^2(t-U)^2} dt$$

After simplification

$$\mu'_{rK} = e^{(r\mu+1/2r^2\sigma^2)} \tag{5.8}$$

The right hand side of the Equation 5.8 is the rth moment expression of a lognormal distribution [40]. Table 5.22 gives μ'_{rK} for different distributions of the random parameter C.

TABLE 5.22 Moments Expressions of Random Parameters K From a Known Distribution of C

No.	Distribution of C ($C = \ell n\, K$)	Probability density function of C $f_Y(y)$	rth moment expression of K μ'_{rK} r = 1, 2, ...
1.	Normal	$\frac{1}{\sqrt{2\lambda\sigma^2}} \exp\left[-\frac{(y-\mu^2)}{2\sigma^2}\right]$	$\exp\left[r\mu + \frac{1}{2} r^2\sigma^2\right]$
2.	Uniform	$\frac{1}{(b-a)}$	$[\exp(rb) - \exp(ra)]/r(b-a)$
3.	Exponential	$\lambda \exp(-\lambda y)$	$\lambda/(\lambda - r)$
4.	Chi-Square	$\frac{\exp(-y/2)\ (x)^{(\lambda-1)}}{2^{\lambda}\Gamma(\lambda)}$	$(1-2q)^{-\lambda/2}$
5.	Gamma	$\theta^{\alpha}\ y^{(\alpha-1)} \exp(-\theta y)/\Gamma(\alpha)$	$(\theta/(\theta - r))^{\alpha}$
6.	Earlong	$(\lambda^{c}\ y^{c-1}\ c^{-\lambda y})/(c-1)!$	$(\lambda/(\lambda - r))^{c}$

5.2.2 Moments Method

In this method the moments of K are obtained from the moments of C, since $K = e^C$ then the rth moment of K is given by

$$
\begin{aligned}
E(K^r) &= E(e^{rC}) \\
&= E\left(\sum_{i=0}^{\infty} \frac{(rC)^i}{i!} \right) \\
&= \sum_{i=0}^{\infty} E\left(\frac{(rC)^i}{i!}\right) \\
&= \sum_{i=0}^{\infty} \frac{r^i}{i!} E(C)^i \\
&= \sum_{i=0}^{\infty} \frac{r^i}{i!} \mu'_{iC}
\end{aligned}
$$

Hence one could approximate the rth moments of K as follows

$$E(K^r) \quad \sum_{i=0}^{n} \frac{r^i}{i!} \mu'_{iC} \tag{5.9}$$

The difficulty of this approach is that a large value of n have to be used in order to achieve close approximation of $E(K^r)$.

In the present development of the surface roughness models, for R_a, m_0, and m_4, the distribution of C in each case has been found to be normal. Therefore, it can be concluded with reference to Equation 5.9, that the distribution of K for the above models is lognormal. For m_2 model, the distribution of C is lognormal with parameter $\theta = 2.7$ and $\sigma^2 = 0.011$. In this case the moments of K ($K=\exp(c)$) can be found by substituting the p.d.f. of the lognormal distribution in Equation 5.7. Since for lognormal distribution:

$$f_C(t) = \frac{1}{\sqrt{2\pi\sigma^2 t}} e^{-1/2\sigma^2(\ell nt - \mu)^2}$$

The rth moment of K is obtained from :

$$\mu'_{rK} = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^\infty \frac{e^{rt}}{t} e^{-1/2\sigma^2(\ell nt - \mu)^2} dt \qquad (5.10)$$

Due to the difficulty in evaluating the integral in Equation 5.10 analytically, the moments μ'_{rK} ($r = 1,2, \ldots$) are found numerically using the Gaussian Loguerre Quadrature formula. The first 8 moments of K are presented in Table 5.24. It can be observed from Table 5.24, that due to the significant deviation of the computed moment ratios from the theoretical moment ratios, none of the seven candidate distributions was found adequate to represent the distribution of K for the m_2 model.

It should be emphasized that even if the distribution of K is none of the well known probability distribution, the linear model represented by Equation 5.2 with parameters C, α, β and γ can still be used for the characterization of the response.

Table 5.23 The Moments of K for the m_2 Model

	r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
μ'_{rK}	$0.160\text{x}10^{4}$	$0.121\text{x}10^{8}$	$0.32\text{x}10^{13}$	$0.98\text{x}10^{18}$	$0.30\text{x}10^{24}$	$0.94\text{x}10^{29}$	$0.29\text{x}10^{35}$	$0.89\text{x}10^{40}$
μ_{rK}	$0.000\text{x}10^{0}$	$0.960\text{x}10^{7}$	$0.31\text{x}10^{13}$	$0.96\text{x}10^{18}$	$0.29\text{x}10^{24}$	$0.91\text{x}10^{29}$	$0.28\text{x}10^{35}$	$0.86\text{x}10^{40}$

μ'_{rK} is the rth momemts about the origin of K
μ_{rK} is the rth momemts about the mean of K

TABLE 5.24 *Moment Ratios of the Theoretical and Computed Moments for K*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	0.1×10^{5}	0.1×10^{9}	0.2×10^{13}				
		c	0.00	0.3×10^{6}	0.1×10^{13}	0.3×10^{19}				
Lognormal	$\ln(\mu'_r/(\mu'_1)^r)$ $/\ln(\mu'_2/\mu'^2_1)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	4.27	7.65	11.02	14.39	17.77	21.15
		c	0.00	0.00	42.33	27.43	10.22	4.00	15.35	24.46
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	0.1×10^{5}	0.1×10^{9}	0.2×10^{13}				
		c	0.00	0.6×10^{6}	0.1×10^{14}	0.2×10^{20}				
Exponential	$\mu'_r/(\mu'_1)^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	4.75	776.93	0.2×10^{6}	0.3×10^{8}	0.6×10^{10}	0.1×10^{13}	0.21×10^{15}
		c	0.00	137.64	0.1×10^{5}	0.6×10^{6}	0.2×10^{8}	0.7×10^{9}	0.2×10^{11}	0.51×10^{12}
Chi-square	$\mu'_r/\mu'_{r-1}-\mu'_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	1.00	6004.8	0.2×10^{6}	0.3×10^{6}	0.3×10^{7}	0.3×10^{8}	0.4×10^{8}	0.4×10^{9}
		c	0.00	3245.8	0.6×10^{7}	0.5×10^{7}	0.4×10^{7}	0.3×10^{7}	0.2×10^{7}	0.5×10^{7}
Gamma	$\mu'_r\mu'_1-\mu'^2_1\mu'_{r-1}$ $/\mu'_{r-1}(\mu'_2-\mu'^2_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	43.29	51.14	51.18	51.18	51.18	51.18
		c	0.00	0.00	2064.73	1604.77	1179.47	923.58	752.98	631.13
Beta	$(\mu'_2-\mu'_1)(\mu'_r/\mu'_{r-1}-\mu'_1)$ $/(\mu'^2_1-\mu'_2)(1-\mu'_r/\mu'_{r-1})$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.25	1.26	1.26	1.28	1.30	1.45
		c	0.00	0.00	37.07	58.00	68.50	74.80	79.80	82.00

a= Theoretical b= Computed c= % Deviation

5.2.3 Estimation of the Distribution Parameters

In Section 5.2.2, the distributions of R_a, m_0, m_2 and m_4 model parameters were identified and listed in Table 5.21. In this section we estimate the parameters of the obtained distributions.

The parameters of a probability distribution could be estimated from its moments. Table 5.25 summarizes the formulae for estimating the parameters of a number of selected distribution from their moments. These formulae were derived from the expressions of the rth moments listed in Table 3.1. The distributions and their corresponding parameters for R_a, m_0, m_2 and m_4 models parameters are summarized in Table 5.26.

5.3 IDENTIFICATION OF THE DISTRIBUTIONS OF RESPONSES R_a, m_0, m_2 and m_4.

In the previous section four surface models have been developed to characterize the random surface profile geometry features R_a, m_0, m_2 and m_4. This section is aimed at the identification of the distributions of these roughness features as well as the distributions of their logarithms, from the experimental data of Chapter 4. The obtained distributions of R_a will be compared with those reported in the

TABLE 5.25 Formulae for Computing the Parameters of the Distributions from their Moments

No.	Distributions	Parameters of the Distributions	
1.	Normal	$\mu = \mu_1'$	$\sigma^2 = \mu_2' - \mu_1'^2$
2.	Lognormal	$\theta = \ln \mu_1' - 0.5\sigma^2$	$\sigma^2 = \ln (\mu_2'/\mu_1')$
3.	Exponential	$\lambda = 1/\mu_1'$	
4.	Chi Square	$\lambda = \mu_1'$	
5.	Gamma	$\alpha = \dfrac{\mu_1'^2}{\mu_2' - \mu_1'^2}$	$\theta = \dfrac{\mu_1'}{\mu_2' - \mu_1'^2}$
6.	Beta	$\alpha = \dfrac{\mu_1' (\mu_2' - \mu_1')}{\mu_1'^2 - \mu_2'}$	$\theta = \alpha/\mu_1' - \alpha$

TABLE 5.26 The Distributions and Parameters of the Surface Royghness Model Parameters

Model	Parameters	Distribution	Distribution Parameters	
	K	Lognormal	$\mu_\ell = 1.325$	$\sigma_\ell^2 = 0.150$
	$C = \ell nK$	Normal	$\mu_C = 1.325$	$\sigma_C^2 = 0.150$
R_a	α	Normal	$\mu = -0.587$	$\sigma^2 = 0.035$
	β	Normal	$\mu = 1.179$	$\sigma^2 = 0.167$
	γ	Normal	$\mu = 0.164$	$\sigma^2 = 0.023$
	K	Lognormal	$\mu_\ell = 3.915$	$\sigma_\ell^2 = 1.150$
	$C = \ell nK$	Normal	$\mu_C = 3.915$	$\sigma_C^2 = 1.150$
m_0	α	Normal	$\mu = 0.125$	$\sigma^2 = 0.003$
	β	Normal	$\mu = -0.017$	$\sigma^2 = 0.25*10^{-3}$
	γ	Normal	$\mu = -0.007$	$\sigma^2 = 0.90*10^{-5}$
	$C = \ell nK$	Lognormal	$\mu_C = 2.076$	$\sigma_C^2 = 0.011$
	α	Normal	$\mu = -0.243$	$\sigma^2 = 0.87*10^{-2}$
m_2	β	Beta	$\alpha = 8.065$	$\theta = 9.190$
	γ	Normal	$\mu = -0.165$	$\sigma^2 = 0.003$
	K	Lognormal	$\mu_\ell = 16.615$	$\sigma_\ell^2 = 1.350$
	$C = \ell nk$	Normal	$\mu_C = 16.615$	$\sigma_C^2 = 1.350$
m_4	α	Normal	$\mu = 0.168$	$\sigma^2 = 0.025$
	β	Normal	$\mu = -0.182$	$\sigma^2 = 0.23*10^{-2}$
	γ	Normal	$\mu = -0.129$	$\sigma^2 = 0.015$

surface roughness characterization literature. The distributions of the surface roughness features will also be compared with the distributions of random parameters of each model in order to check the validity of the models.

To identify the distributions of each surface profile feature R_a, m_0, m_2 and m_4 as well as the distributions of their logarithms; $\ell n\ R$, $\ell n\ m_0$, $\ell n\ m_2$ and $\ell n\ m_4$, the first 8 moments were estimated from the experimental data of each feature. The estimated moments were then used to obtain the moment ratios presented in Table 5.27 to Table 5.34. The criterion of minimum percent deviation of the computed moment ratios from the theoretical moments ratios were used to identify an adequate distribution that represents each surface roughness features.

The moment ratios for R_a, $\ell n\ R_a$, m_0, $\ell n\ m_0$, m_4 and $\ell n\ m_4$ are presented in Tables 5.27 to 5.34 respectively. By comparing the theoretical and estimated moment ratios for each profile characterisitic parameter, the distribution of R_a, m_0 and m_4 were found to be lognormal, whereas the distributions of $\ell n\ R_a$, $\ell n\ m_0$ and $\ell n\ m_4$ were found to be normal.

The moment ratios for m_2 and $\ell n\ m_2$ are presented in Table 5.31 and 5.32 respectively. By comparing the estimated and theoretical moment ratios for m_2, and $\ell n\ m_2$ it was found that gamma and lognormal distributions have minimum

TABLE 5.27 Moment Ratios of the Theoretical and Computed Moments for Respose R_a

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	69.44	$0.2\text{x}10^5$	$0.1\text{x}10^8$				
		c	0.00	2214.7	$0.2\text{x}10^6$	$0.9\text{x}10^5$				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$ $/\ln(\mu_2'/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	3.10	6.25	10.19	14.25	19.38	25.55
		c	0.00	0.00	3.33	4.16	1.97	4.96	7.71	8.75
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	69.44	$0.2\text{x}10^5$	$0.1\text{x}10^8$				
		c	0.00	3709.1	$0.6\text{x}10^6$	$0.4\text{x}10^7$				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	2.12	10.56	120.26	2168.67	4611.34	109876.9	376543.9
		c	0.00	6.20	76.08	401.09	1707.20	6304.73	20486.75	59165.65
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	22.90	80.97	211.58	347.09	412.93	438.12	448.93
		c	0.0	1045.32	1924.42	3426.45	4238.67	4029.28	3551.04	3106.67
Gamma	$\mu_r'\mu_1'-\mu_1'^2\mu_{r-1}$ $/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	3.53	9.24	15.15	18.02	19.23	19.59
		c	0.00	0.00	76.75	707.90	278.81	260.53	218.78	179.98
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$ $/(\mu_1^2-\mu_2)(1-\mu_r/\mu_{r-1})$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.49	1.69	1.75	1.76	1.77	1.77
		c	0.00	0.00	25.52	43.63	56.29	64.73	70.53	74.70

a= Theoretical b= Computed c= % Deviation

TABLE 5.28 Moment Ratios of the Theoretical and Computed Moments for Respose In R_a

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	2.89	13.91	98.94				
		c	0.00	3.64	7.23	5.77				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$ $/\ln(\mu_2'/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.77	5.20	8.17	11.64	15.53	19.81
		c	0.00	0.00	7.37	13.30	18.23	22.43	26.06	29.23
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	2.89	13.91	98.94				
		c	0.00	60.59	261.42	999.34				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.10	1.31	1.64	2.18	3.04	4.40	6.63
		c	0.00	44.99	78.27	93.15	98.18	99.57	99.91	99.98
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.26	0.49	0.69	0.87	1.04	1.19	1.34
		c	0.0	86.70	87.72	88.50	89.11	89.62	90.05	90.41
Gamma	$\mu_r'\mu_1'-\mu_1'^2\mu_{r-1}'$ $/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.85	2.59	3.27	3.90	4.49	5.05
		c	0.00	0.00	7.62	13.38	18.04	21.89	25.13	27.85
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$ $/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.65	2.13	2.50	2.78	3.03	3.23
		c	0.00	0.00	17.32	29.08	37.69	44.31	49.54	53.78

a= Theoretical b= Computed c= % Deviation

TABLE 5.29 *Moment Ratios of the Theoretical and Computed Moments for Respose* m_0

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	118.26	58636.8	$0.36\text{x}10^8$				
		c	0.0	3842.24	$0.39\text{x}10^6$	$0.345\text{x}10^6$				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	b	0.00	1.00	2.97	5.69	9.10	13.61	18.77	24.91
		c	0.00	0.00	0.91	5.16	9.00	9.27	10.62	11.04
	$/\ln(\mu_2'/\mu_1'^2)$	a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}/(\mu_2)^r$	b	1.00	118.26	58636.8	$0.36\text{x}10^8$				
		c	0.00	$0.64\text{x}10^4$	$0.15\text{x}10^7$	$0.39\text{x}10^6$				
		a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
Exponential	$\mu_r'/(\mu_1')^r$	b	1.00	3.18	31.24	672.45	20635.0	72426.6	0.36x10	$0.89\text{x}10^6$
		c	0.00	59.15	420.68	2701.87	17096.3	100493.0	0.53x10	$0.25\text{x}10^7$
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	b	0.00	227.4	918.36	2138.75	3092.81	3552.34	3755.85	3854.05
		c	0.0	11271.0	22859.0	35537.50	38560.22	35423.9	31198.79	27428.9
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	4.04	9.40	13.59	15.62	16.52	16.95
	$/\mu_{r-1}'(\mu_2' - \mu_1'^2)$	c	0.00	0.00	101.9	213.40	239.98	212.40	175.75	142.09
Beta	$(\mu_2' - \mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.30	1.38	1.41	1.41	1.41	1.42
	$/(\mu_1'^2 - \mu_2')(1 - \mu_r'/\mu_{r-1}')$	c	0.00	0.00	34.65	53.77	64.83	71.74	76.41	79.77

a= Theoretical b= Computed c= % Deviation

TABLE 5.30 Moment Ratios of the Theoretical and Computed Moments for Respose In m_0

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	2.89	13.59	96.12				
		c	0.0	3.52	9.34	8.46				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.80	5.34	8.47	12.15	16.33	20.97
	$/\ln(\mu_2'/\mu_1'^2)$	c	0.00	0.00	5.98	11.00	15.28	18.99	22.23	25.09
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	2.89	13.59	96.12				
		c	0.00	60.80	253.22	968.00				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.07	1.21	1.43	1.76	2.26	2.99	4.08
		c	0.00	46.53	79.86	94.04	98.52	99.68	99.94	99.98
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.28	0.53	0.75	0.95	1.14	1.32	1.49
		c	0.0	85.86	86.77	87.49	88.09	88.59	89.00	89.37
Gamma	$\mu_r'\mu_1' - \mu_1'^2 \mu_{r-1}'$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.87	2.65	3.37	4.04	4.66	5.26
	$/\mu_{r-1}'(\mu_2' - \mu_1'^2)$	c	0.00	0.00	6.40	11.54	15.76	19.28	22.25	24.78
Beta	$(\mu_2' - \mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.74	2.33	2.81	3.21	3.56	3.87
	$/(\mu_1'^2 - \mu_2')(1 - \mu_r'/\mu_{r-1}')$	c	0.00	0.00	12.81	22.36	29.78	35.72	40.59	44.67

a= Theoretical b= Computed c= % Deviation

TABLE 5.31 Moment Ratios of the Theoretical and Computed Moments for Respose m_2

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Normal	$\mu_{2r}/(\mu_2)^r$	a	1.00	3.00	15.00	105.00				
		b	1.00	3.07	15.29	100.65				
		c	0.00	2.35	1.94	4.15				
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$ $/\ln(\mu_2'/\mu_1'^2)$	a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
		b	0.00	1.00	2.97	5.88	9.68	14.33	14.75	25.89
		c	0.00	0.00	0.89	1.94	3.13	4.46	4.46	5.92
Uniform	$\mu_{2r}/(\mu_2)^r$	a	1.00	1.80	3.85	9.00				
		b	1.00	3.07	15.29	100.65				
		c	0.00	70.58	297.17	1018.31				
Exponential	$\mu_r'/(\mu_1')^r$	a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
		b	1.00	1.04	1.13	1.27	1.49	1.81	2.26	2.91
		c	0.00	47.89	81.16	94.68	98.75	99.75	99.95	99.99
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1$	a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
		b	0.00	0.41	0.83	1.25	1.67	2.07	2.46	2.83
		c	0.0	79.30	79.16	79.10	79.13	79.25	79.46	79.76
Gamma	$\mu_r'\mu_1'-\mu_1'^2\mu_{r-1}'$ $/\mu_{r-1}'(\mu_2'-\mu_1^2)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	2.01	3.03	4.03	5.01	5.95	6.85
		c	0.00	0.00	0.67	0.97	0.85	0.28	0.75	2.22
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$ $/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.93	2.77	3.55	4.25	4.87	5.40
		c	0.00	0.00	3.70	7.45	11.23	15.02	18.79	22.53

a= Theoretical b= Computed c= % Deviation

TABLE 5.32 Moment Ratios of the Theoretical and Computed Moments for Respose In m_2

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	2.73	12.36	1162.76				
		c	0.00	9.04	17.60	1007.39				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	b	0.00	1.00	2.97	5.88	9.71	14.41	19.97	26.35
	$/\ln(\mu_2'/\mu_1'^2)$	c	0.00	0.00	0.97	1.95	2.93	3.91	4.89	5.87
		a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}/(\mu_2)^r$	b	1.00	2.73	12.36	1162.76				
		c	0.00	51.59	221.02	12819.0				
		a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
Exponential	$\mu_r'/(\mu_1')^r$	b	1.00	1.00	1.02	1.05	1.08	1.12	1.18	1.24
		c	0.00	49.58	82.92	95.63	99.09	99.84	99.97	99.99
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	b	0.00	0.018	0.037	0.05	0.072	0.09	0.10	0.12
		c	0.0	99.07	99.08	99.09	99.10	99.11	99.12	99.13
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.98	2.93	3.87	4.78	5.66	6.52
	$/\mu_{r-1}'(\mu_2'-\mu_1^2)$	c	0.00	0.00	1.06	2.16	3.29	4.43	5.59	6.76
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1)$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.90	2.85	3.70	4.50	5.30	6.04
	$/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	c	0.00	0.00	2.45	4.83	7.15	9.39	11.57	13.67

a= Theoretical b= Computed c= % Deviation

TABLE 5.33 *Moment Ratios of the Theoretical and Computed Moments for Respose m_4*

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	18.25	987.16	66984.34				
		c	0.00	508.52	6481.10	63699.37				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	b	0.00	1.00	2.96	5.83	9.53	13.96	19.39	25.12
	$/\ln(\mu_2'/\mu_1'^2)$	c	0.00	0.00	1.42	2.83	4.70	6.93	7.67	10.28
		a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}/(\mu_2)^r$	b	1.00	18.25	987.16	66989.34				
		c	0.00	914.19	25540.6	744225.9				
		a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
Exponential	$\mu_r'/(\mu_1')^r$	b	1.00	1.70	4.83	18.19	100.30	895.30	6275.85	53619.70
		c	0.00	14.78	19.34	24.20	16.41	24.35	24.52	32.98
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	b	0.00	10.41	27.19	49.95	71.50	86.28	94.98	99.94
		c	0.0	420.74	579.98	732.53	793.80	762.80	691.51	613.92
Gamma	$\mu_r'\mu_1' - \mu_1'^2\mu_{r-1}'$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	2.61	4.79	6.86	9.38	11.11	14.59
	$/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	c	0.00	0.00	30.58	59.87	71.64	87.60	85.16	108.42
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.54	1.82	1.95	2.00	2.03	2.04
	$/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	c	0.00	0.00	22.89	32.29	51.29	59.93	66.18	70.83

a= Theoretical b= Computed c= % Deviation

TABLE 5.34 Moment Ratios of the Theoretical and Computed Moments for Respose In m_4

Distribution	Moments Ratio		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
		a	1.00	3.00	15.00	105.00				
Normal	$\mu_{2r}/(\mu_2)^r$	b	1.00	2.91	13.88	91.05				
		c	0.00	3.00	7.46	13.28				
		a	0.00	1.00	3.00	6.00	10.00	15.00	21.00	28.00
Lognormal	$\ln(\mu_r'/(\mu_1')^r)$	b	0.00	1.00	2.76	5.13	8.03	11.38	15.16	19.30
	$/\ln(\mu_2'/\mu_1'^2)$	c	0.00	0.00	8.03	14.43	19.68	24.08	27.83	31.07
		a	1.00	1.80	3.85	9.00				
Uniform	$\mu_{2r}/(\mu_2)^r$	b	1.00	2.91	13.88	91.05				
		c	0.00	61.66	260.52	911.67				
		a	1.00	2.00	6.00	24.00	120.00	720.00	5040.00	40320.00
Exponential	$\mu_r'/(\mu_1')^r$	b	1.00	1.09	1.28	1.59	2.06	2.79	3.93	5.71
		c	0.00	45.27	78.62	93.37	98.28	99.61	99.92	99.98
		a	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00
Chi-square	$\mu_r'/\mu_{r-1}' - \mu_1'$	b	0.00	0.23	0.42	0.58	0.72	0.85	0.98	1.09
		c	0.0	88.56	89.58	90.35	90.95	91.43	91.82	92.15
Gamma	$\mu_r'\mu_1'-\mu_1'^2\mu_{r-1}'$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.82	2.53	3.16	3.75	4.29	4.80
	$/\mu_{r-1}'(\mu_2'-\mu_1'^2)$	c	0.00	0.00	8.91	15.62	20.87	25.08	28.52	31.37
Beta	$(\mu_2'-\mu_1')(\mu_r'/\mu_{r-1}' - \mu_1')$	a	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
		b	0.00	1.00	1.635	2.08	2.43	2.71	2.95	3.40
	$/(\mu_1'^2-\mu_2')(1-\mu_r'/\mu_{r-1}')$	c	0.00	0.00	18.23	30.41	39.15	45.75	50.91	55.07

a= Theoretical b= Computed c= % Deviation

percent deviations respectively. Table 5.35 summarizes the results.

Olsen [41] and Allen [42] have concluded that for a fine turning process the R_a values are normally distributed, while Petropouls [43] and Sundaram [44] reported they do not. These conflicting conclusions are attributed to the difficulty of reproducing results of experimental work conducted at different laboratories at different operational variables.

A fairly comprehensive survey of the literature indicates the lack of any reference to the distributions of m_0, m_2 and m_4 of the fine turning surfaces. Therefore the distributions of m_0, m_2 and m_4 given in Table 5.35 are reported for the first time.

The resulting distributions of R_a, m_0, m_2 and m_4 and their logarithms can be verified from the distributions of their corresponding model parameters listed in Table 5.24. As the model represented by Equation 5.2 is linear, it is possible to use the reproductive property of the moment generating function (Equation 3.13) to show that when the model parameters C, α, β and γ are normally distributed then the dependent variable Y should also be normally distributed. This also leads to the conclusion that for the empirical model represented by Equation 5.1, the response

TABLE 5.35 Adequate Distributions of Surface Profile Characteristic Parameters

Characteristic Parameters	Adequate Distributions
R_a	Lognormal
$\ln R_a$	Normal
m_0	Lognormal
$\ln m_0$	Normal
m_2	Gamma
$\ln m_2$	Lognormal
m_4	Lognormal
$\ln m_4$	Normal

R = exp(Y) should be lognormally distributed when Y is normally distributed as shown in Table 5.22.

Since Table 5.26 indicates that C, α, β and γ of the R_a, m_0, and m_4 models are all normally distributed, it can be concluded that the distributions of $\ell n\ R_a$, $\ell n\ m_0$ and $\ell n\ m_4$ are also represented by normal distribution. This result is confirmed by Table 5.35, which shows the adequate distributions of $\ell n\ R_a$, $\ell n\ m_0$ and $\ell n\ m_4$ to be normally distributed. Table 5.35 confirms the fact that the distributions of R_a, m_0 and m_4 have to be represented by a lognormal distribution, because these are the exponents of the respective responses.

5.4 RANDOM MODELS Vs CONVENTIONAL MODELS

As discussed in Chapter 1, the conventional least square technique, estimates the mean values of the model parameters only , which results in a constant value of the response at fixed levels of process variables, inspite of the fact that the response is not constant due to the inherent variation in the process. On the other hand, the variation in the response is fully captured by using the statistical modeling technique which treats the model parameters as random variables and characterise their probability distributions. The difference between the conventional and the proposed statistical modeling technique will be highlighted below by comparing the developed surface roughness models with the conventional models obtained from the experimental data. The surface roughness data generated in Chapter 4 were used for developing the R_a, m_0, m_2 and m_4 response models using the conventional least square estimation technique and the parameters obtained are listed in Table 5.36 together with the mean values (first moments about origin) of the respective parameters obtained from the statistical modeling technique.

A closer examination of Table 5.36 indicates that each of the model parameters C, α, β and γ estimated using the least square technique and computed from the statistical modeling technique varies between 1.7 to 7.3 percent, 0.6 to 11.8 percent, 1.89 to 12 percent and 0.4 to 6.6 percent for R_a, m_0, m_2 and m_4 models respectively.

The observed variations are assumed to be small and at this stage it is possible to conclude that both techniques gave approximately the same estimate of the mean values of the model parameters. Since the conventional technqiue is limited to the estimation of the mean values of the model parameters only the random nature of the response is not expected to be reflected by the conventional models. The statistical modeling technique on the other hand has the ability to identify the adequate distributions of the random parameters of the models.

TABLE 5.36 The Random Models Vs. Conventional Models

Response	Parameters Estimated By	Model Parameters C	α	β	γ
R_a	Conventional Method	1.287	-0.570	1.159	0.152
	Statistical Technique	1.325	-0.587	1.179	0.164
	% Deviation	2.870	2.890	1.690	7.320
m_0	Conventional Method	3.940	0.129	-0.019	-0.008
	Statistical Technique	3.915	0.125	-0.017	-0.007
	% Deviation	0.640	3.200	11.760	9.950
m_2	Conventional Method	9.090	-0.234	0.463	-0.188
	Statistical Technique	8.019	-0.243	0.467	-0.165
	% Deviation	11.770	3.700	1.800	12.230
m_4	Conventional Method	16.678	0.173	-0.194	-0.128
	Statistical Technique	16.615	0.168	-0.182	-0.129
	% Deviation	0.370	2.970	6.590	0.770

The variation in the responses due to the variation in the model parameters at a fixed level of process variables can be observed graphically. The variation in responses, R_a, m_0, m_2 and m_4, due to the variation in an individual model parameter while keeping the other model parameters constant at their mean values, are plotted in Fig. 5.1 to 5.16 at each of the test conditions of Table 4.1. For example, Fig. 5.1 shows the variation in response R_a (at each of the 9 test conditions of Table 4.1), due to the variation in model parameter K, while α, β and γ are kept constant at their mean values.

The conventional technique uses only the mean values of the model parameters, which results in only one response of R_a, as shown by the dotted line on Fig. 5.1. For example, the value of R_a is 8.0, 6.9 and 3.4 μm at test-1, 2, and 3 respectively. On the other hand, the statistical modeling technique shows that R_a varies between 1.4 to 12.0 μm, 1.4 to 12.1 μm and 1.4 to 6.10 μm at test-1, 2, and 3 respectively, while the variation of K is between 1.00 to 9.00. Similarly, the variation in m_0, m_2, and m_4 due to the variation in model parameters K, α, β and γ can be observed from Fig. 5.5 to 5.16.

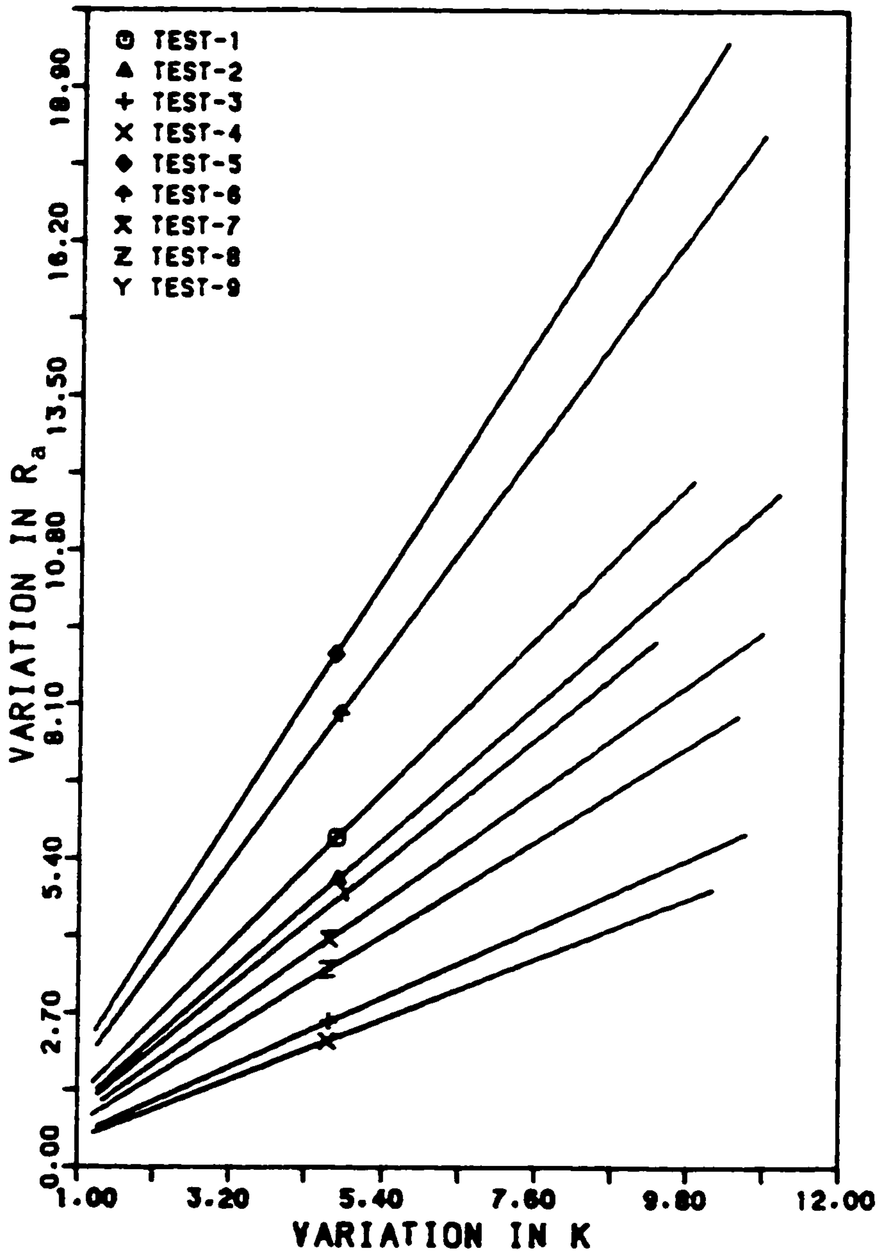


Figure 5.1. Variation in R_a with Respect to Parameter K

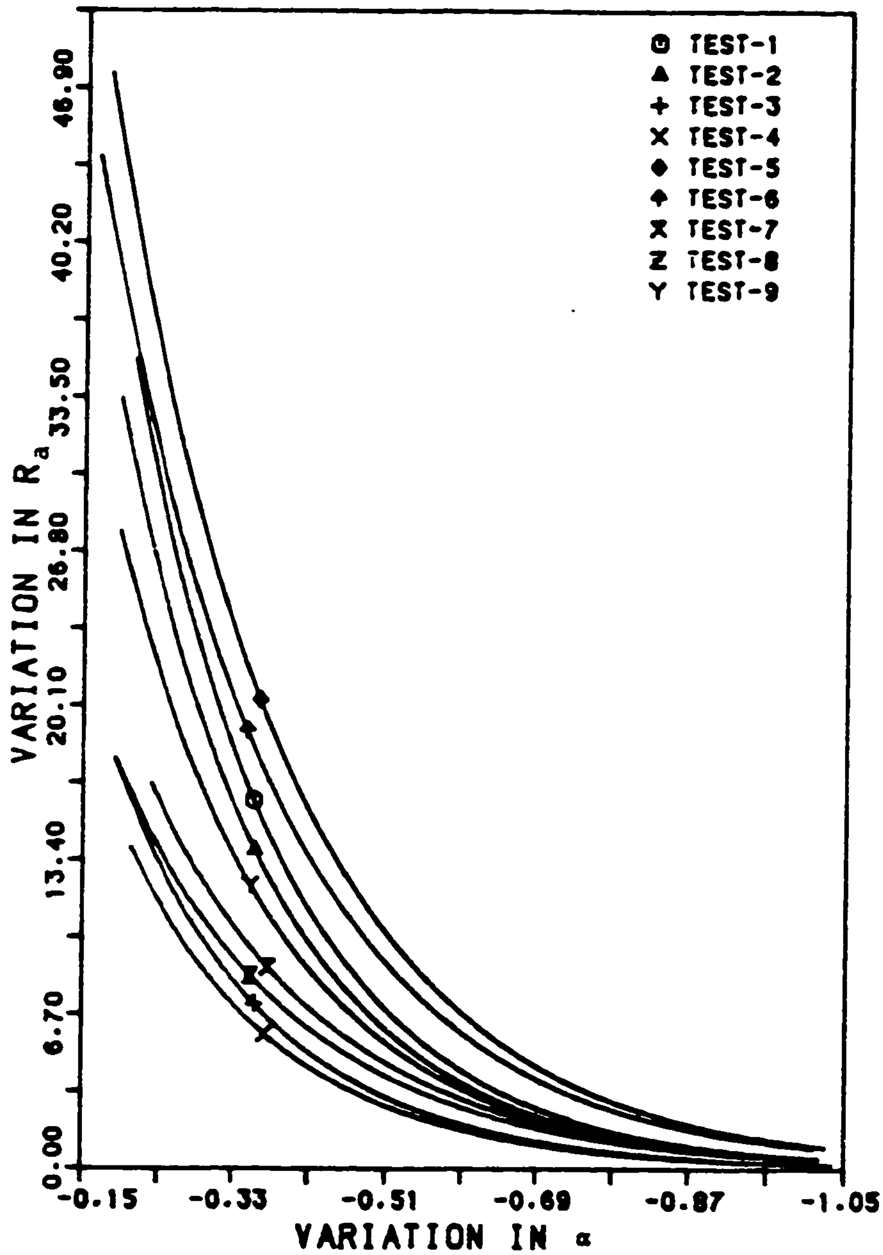


Figure 5.2. Variation in R_a with Respect to Parameter α

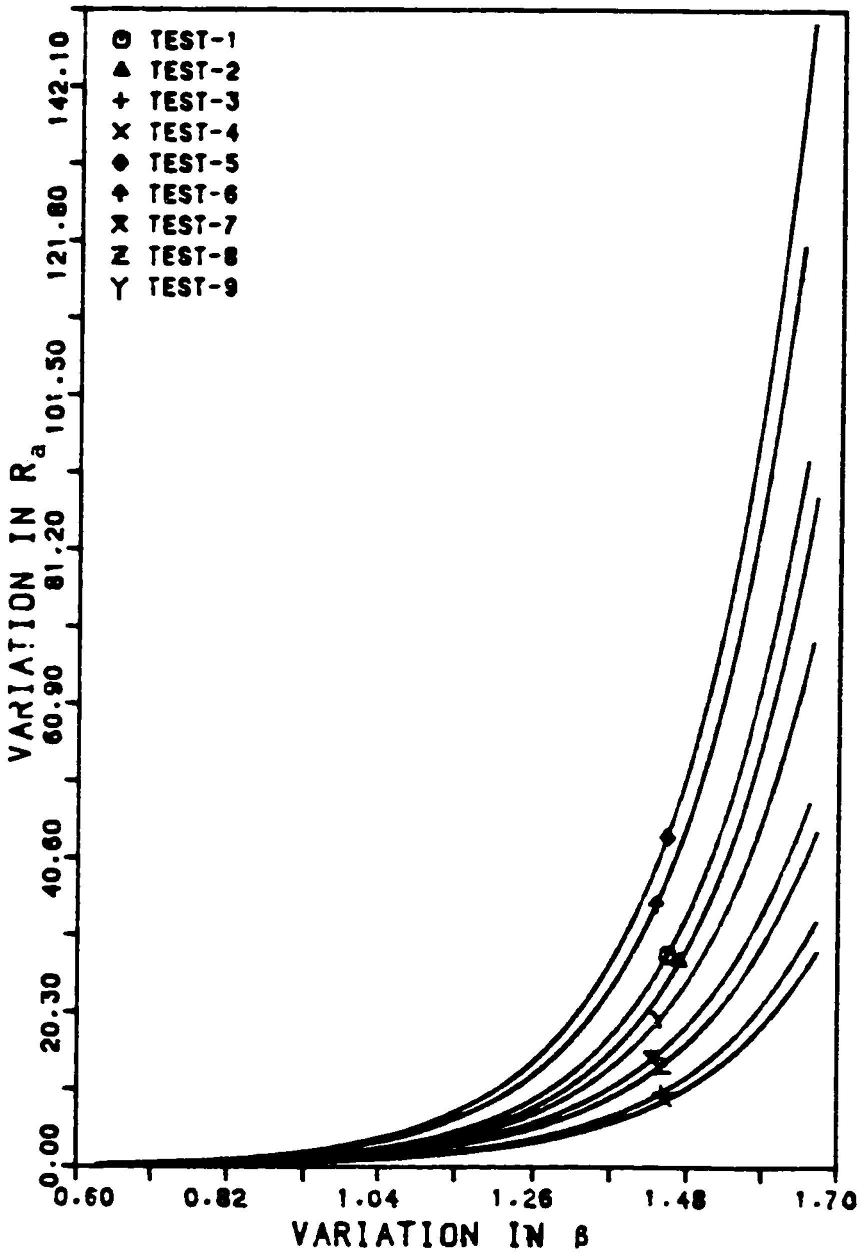


Figure 5.3. Variation in R_a with Respect to Parameter β

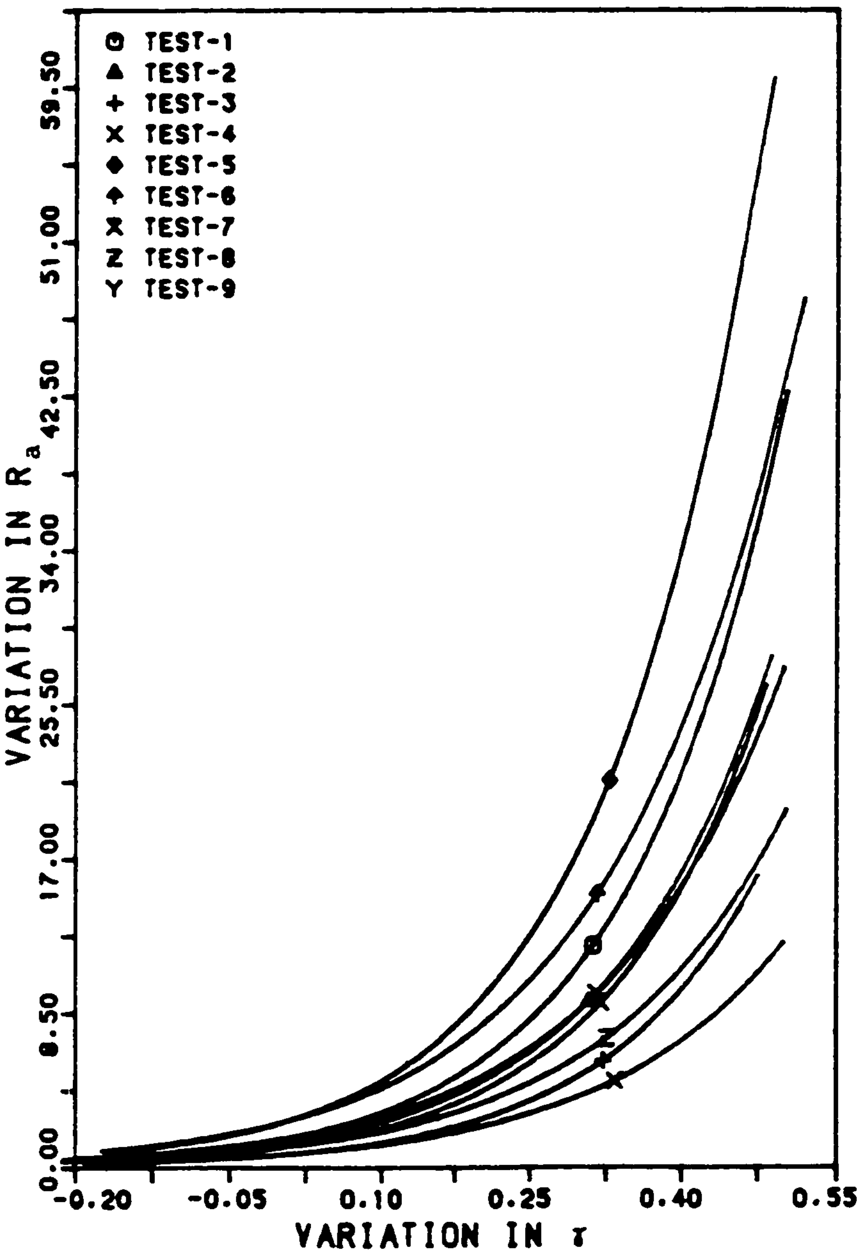


Figure 5.4. Variation in R_a with Respect to Parameter γ

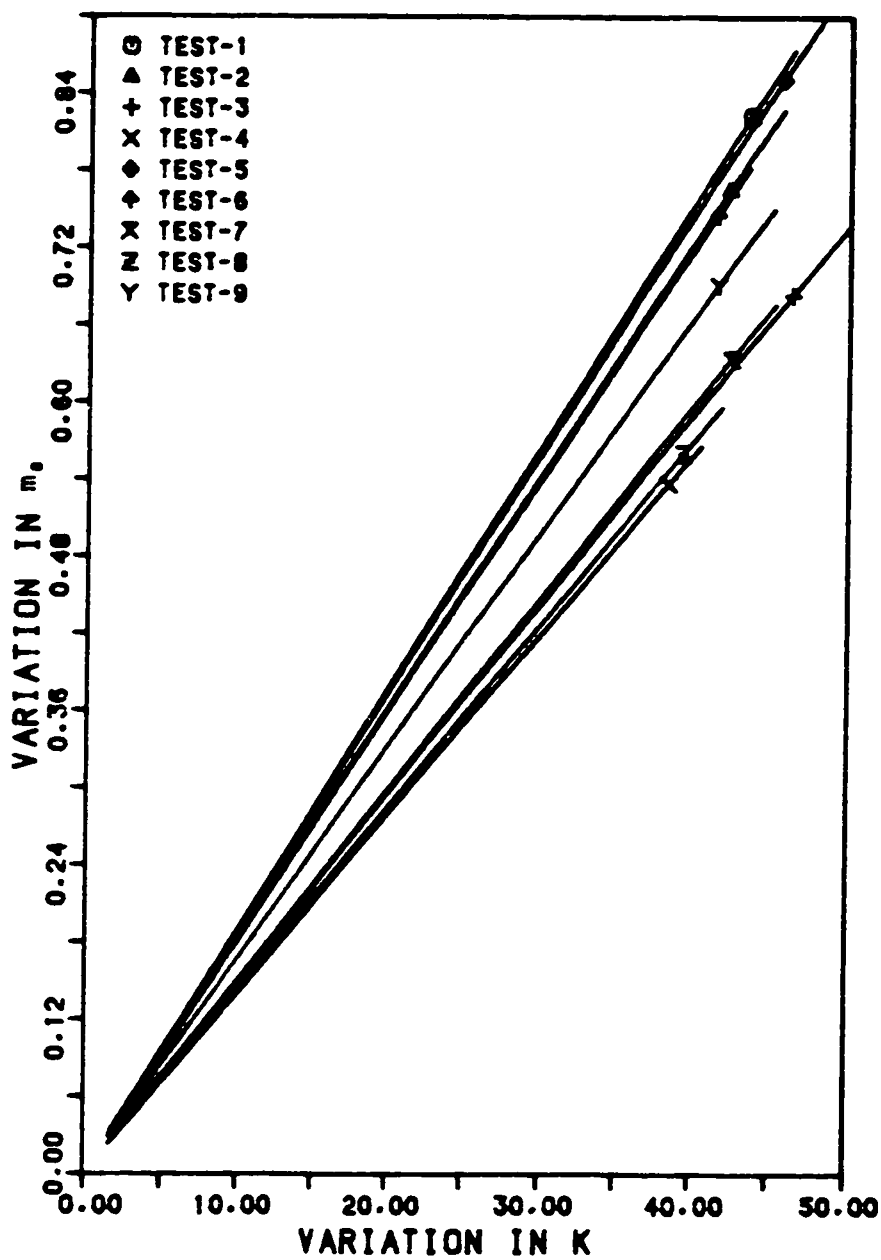


Figure 5.5. Variation in m_e with Respect to Parameter K

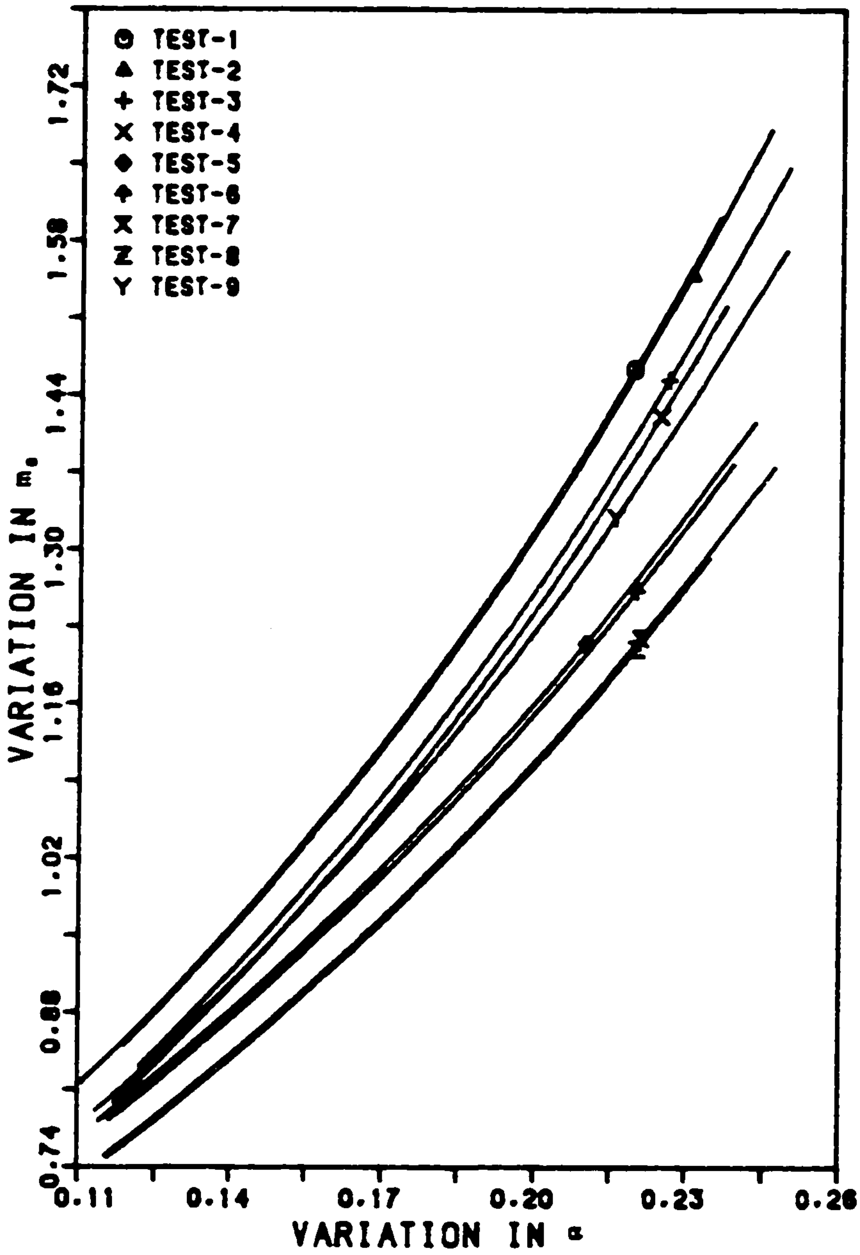


Figure 5.6. Variation in m_e with Respect to Parameter α

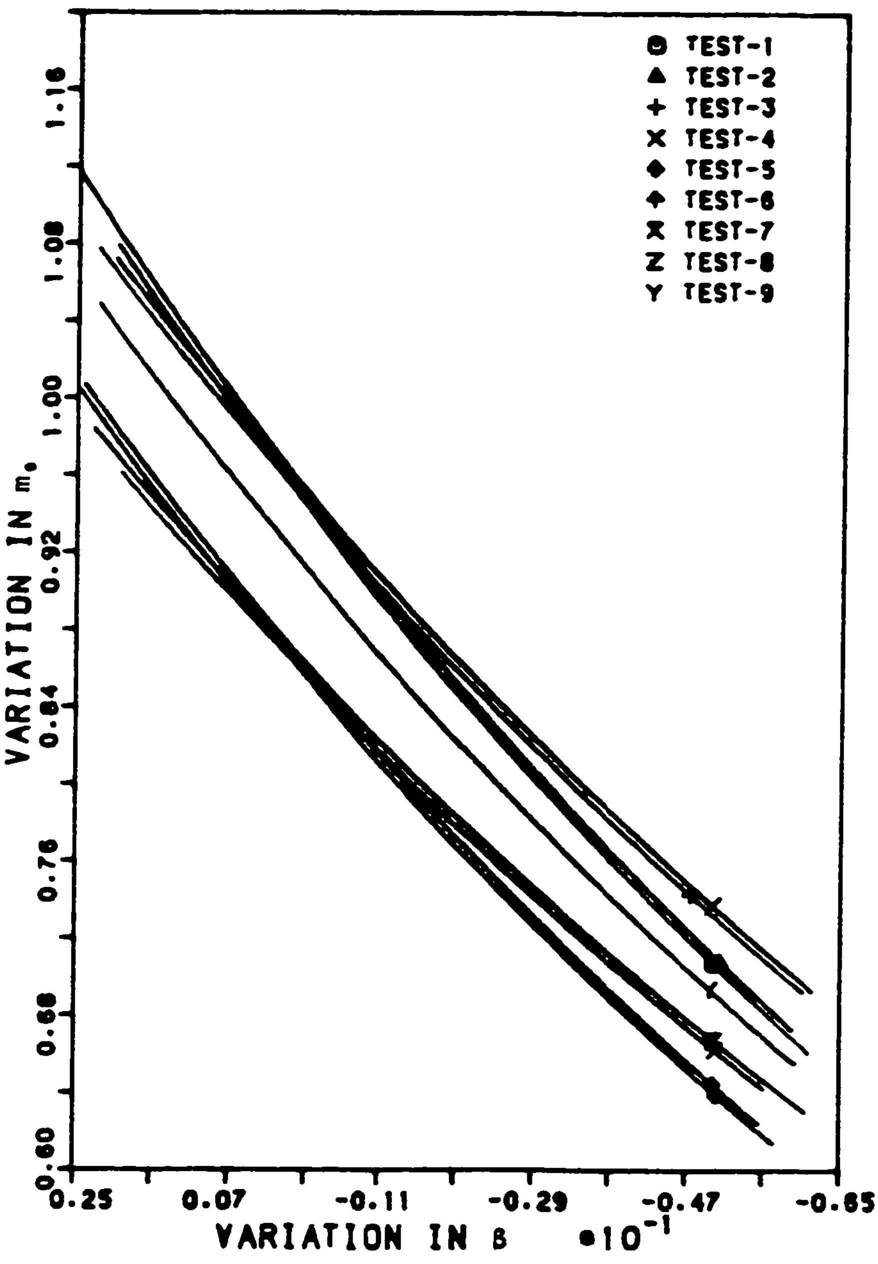


Figure 5.7. Variation in m_e with Respect to Parameter β

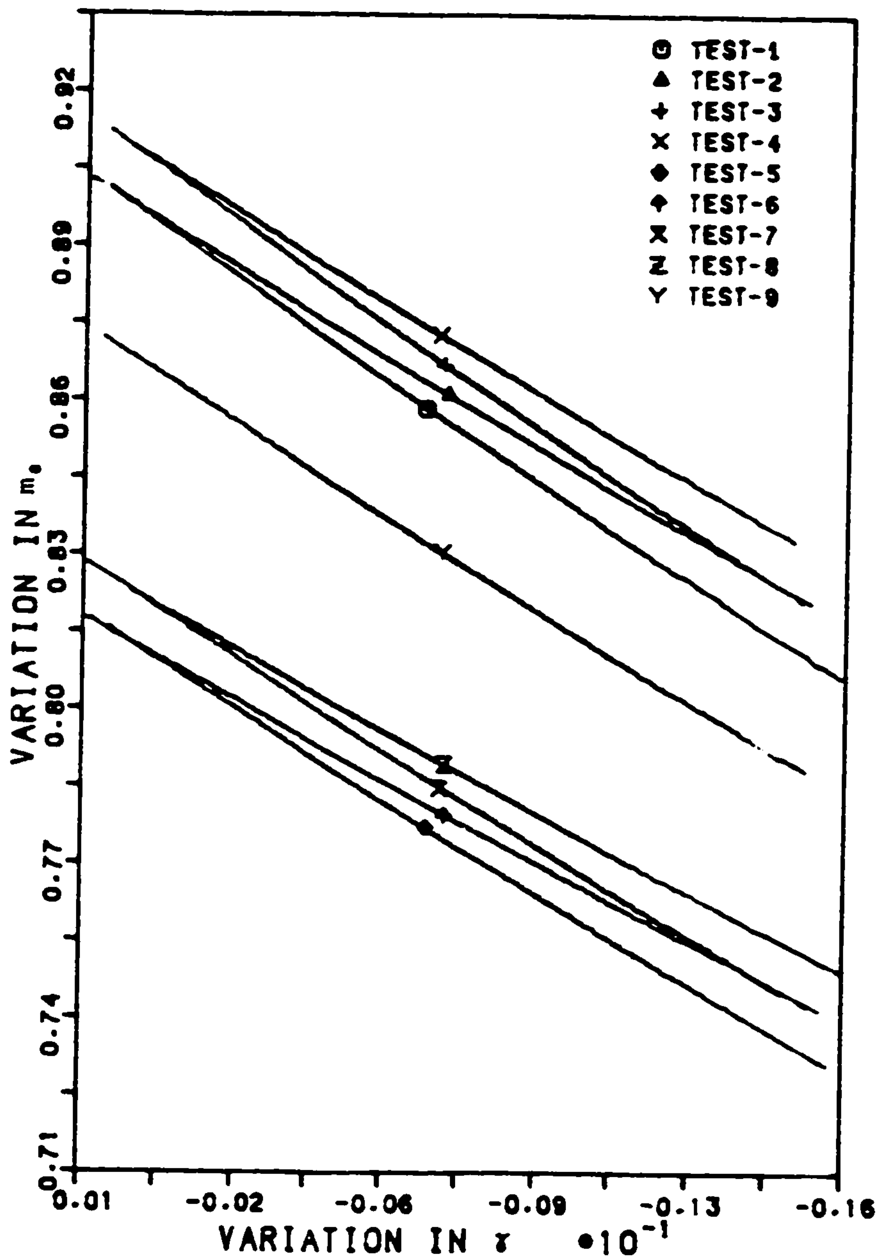


Figure 5.8. Variation in m_e with Respect to Parameter γ

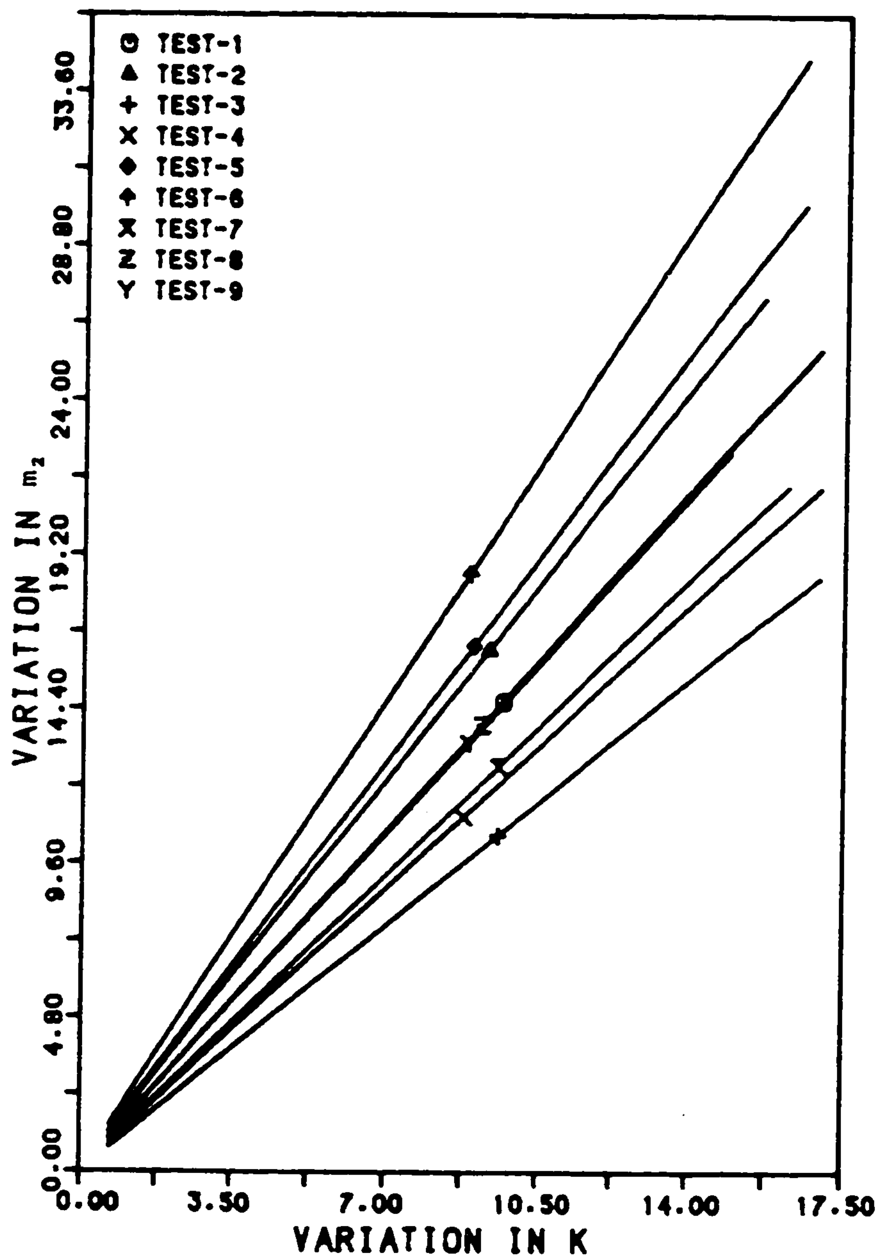


Figure 5.9. Variation in m_2 with Respect to Parameter K

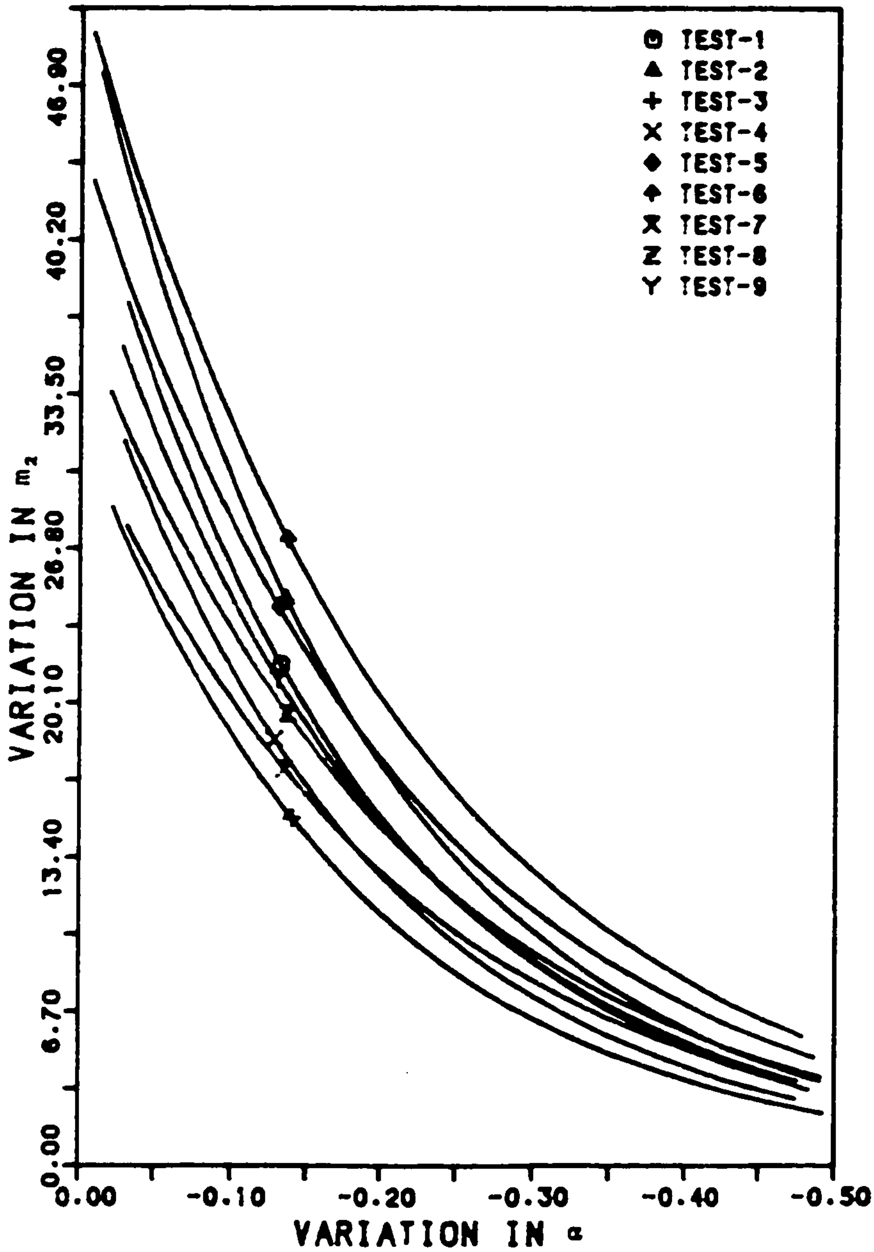


Figure 5.10. Variation in m_2 with Respect to Parameter α

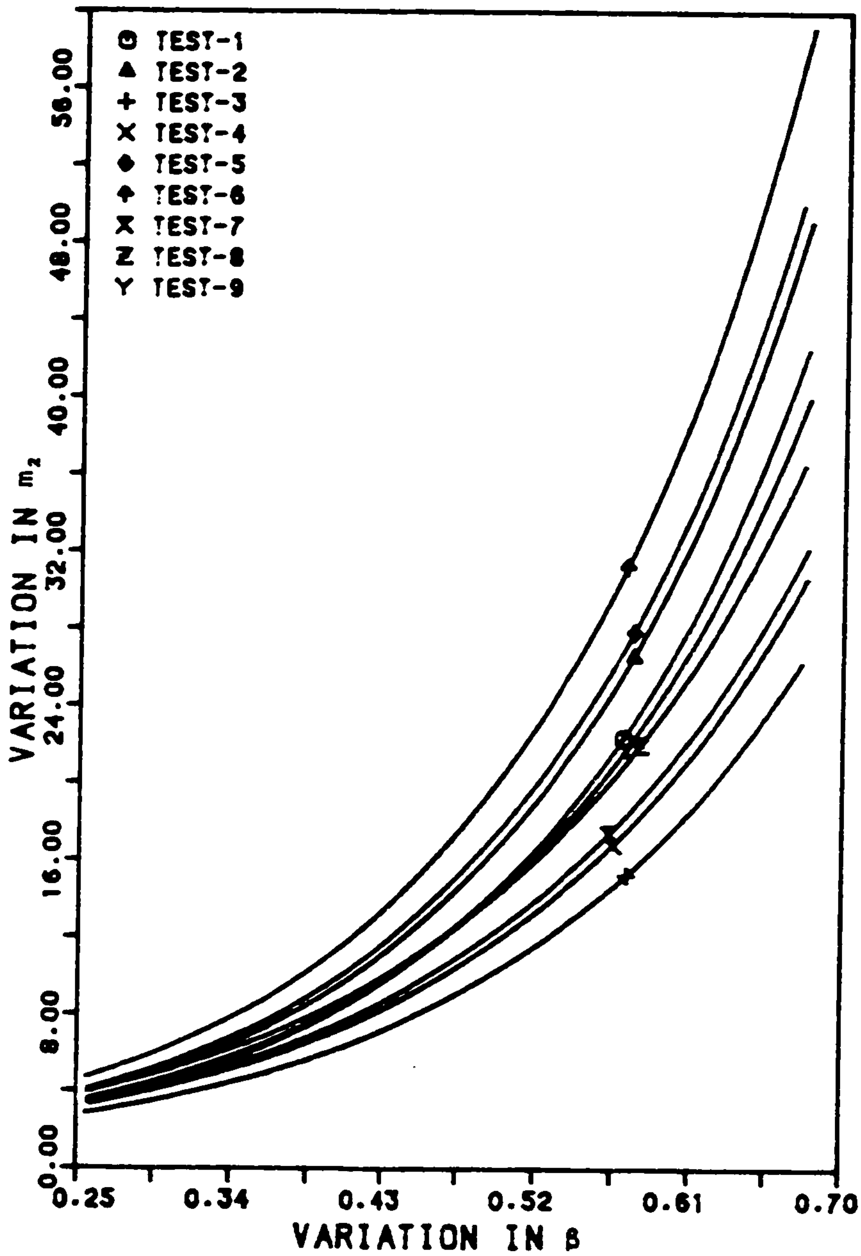


Figure 5.11. Variation in m_2 with Respect to Parameter β

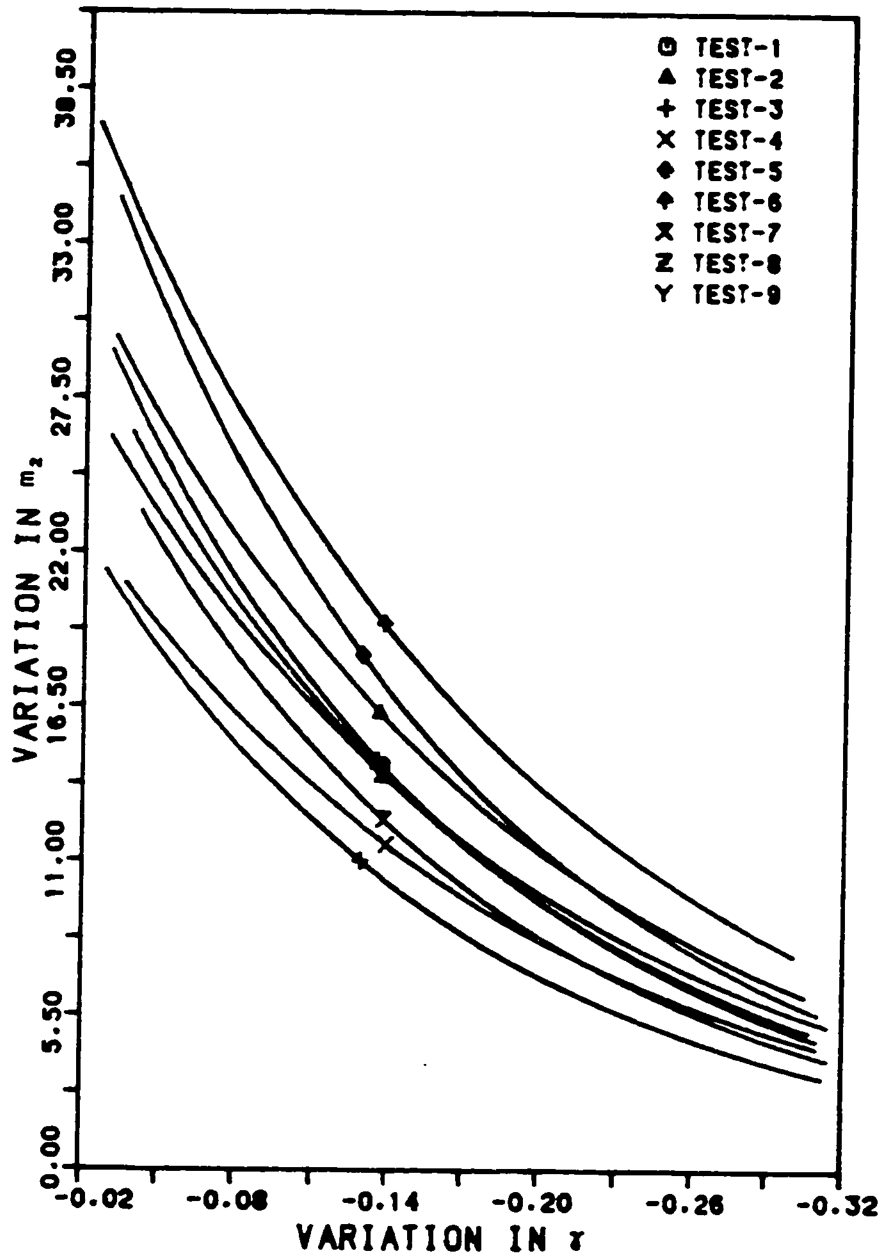


Figure 5.12. Variation in m_2 with Respect to Parameter γ

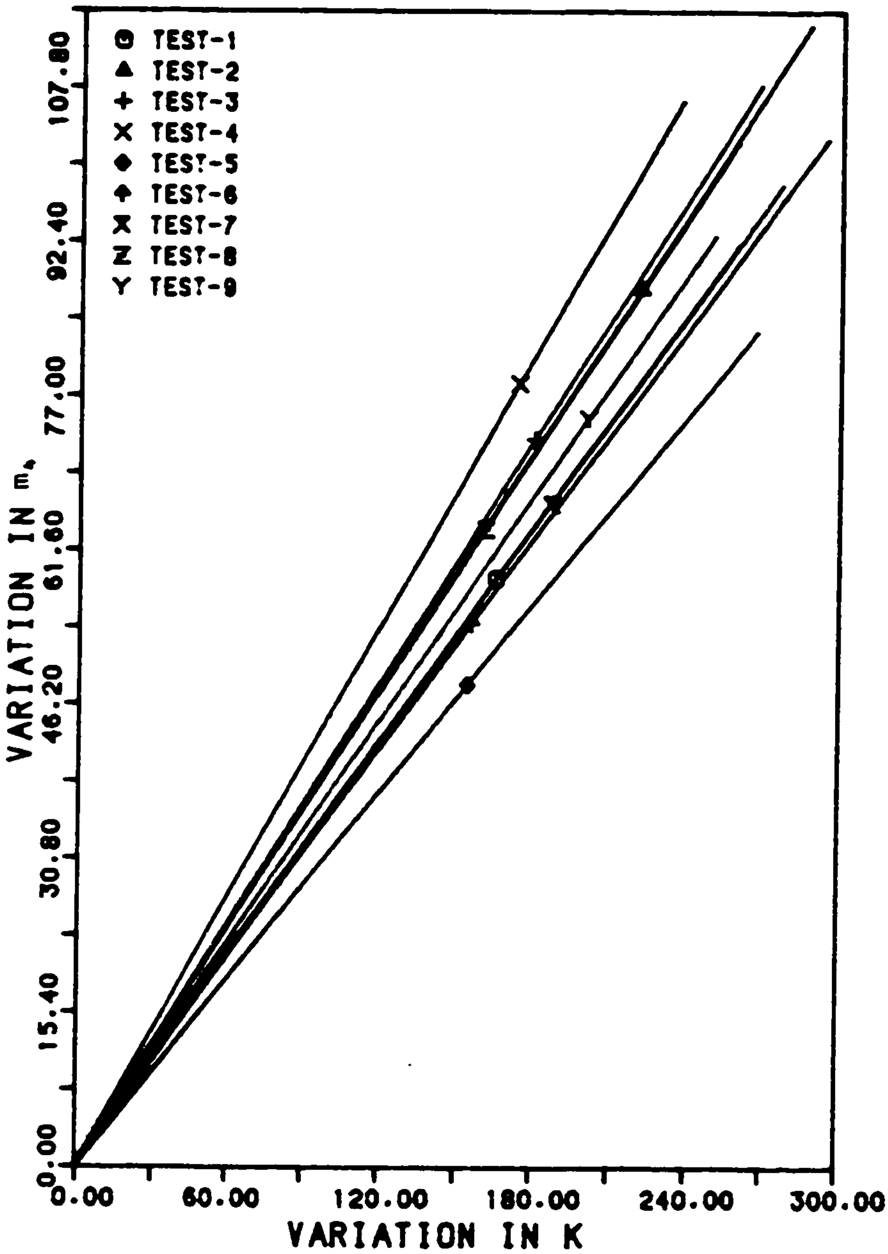


Figure 5.13. Variation in m_4 with Respect to Parameter K

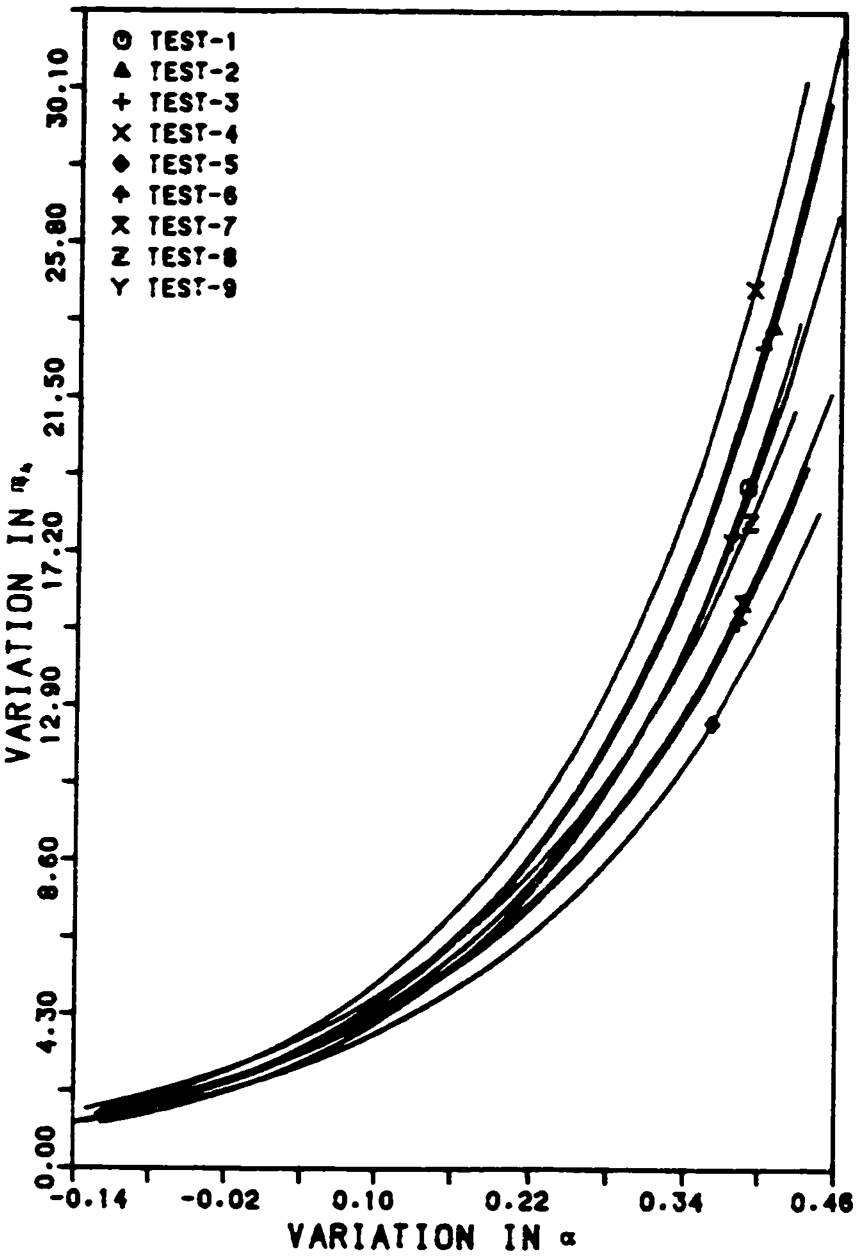


Figure 5.14. Variation in m_4 with Respect to Parameter α

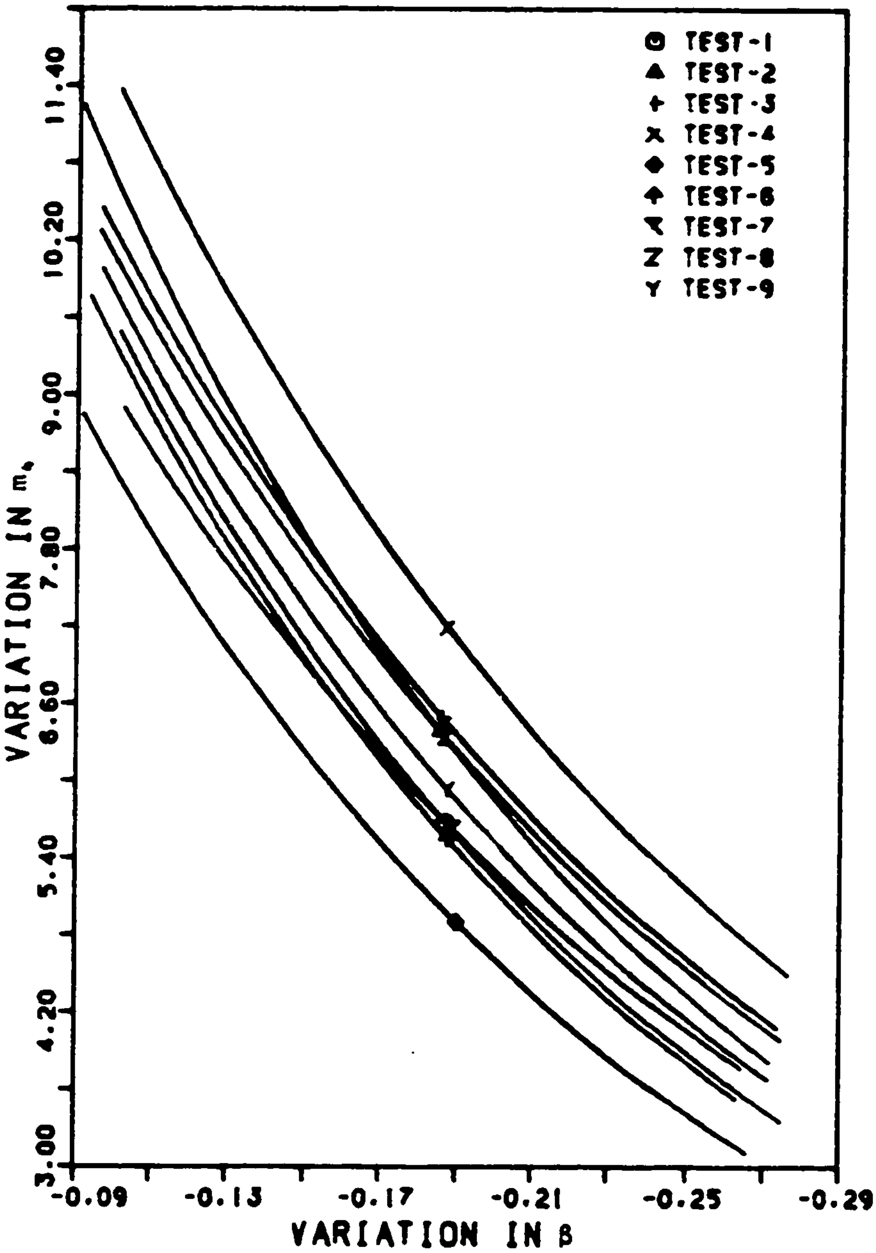


Figure 5.15. Variation in m_4 with Respect to Parameter β

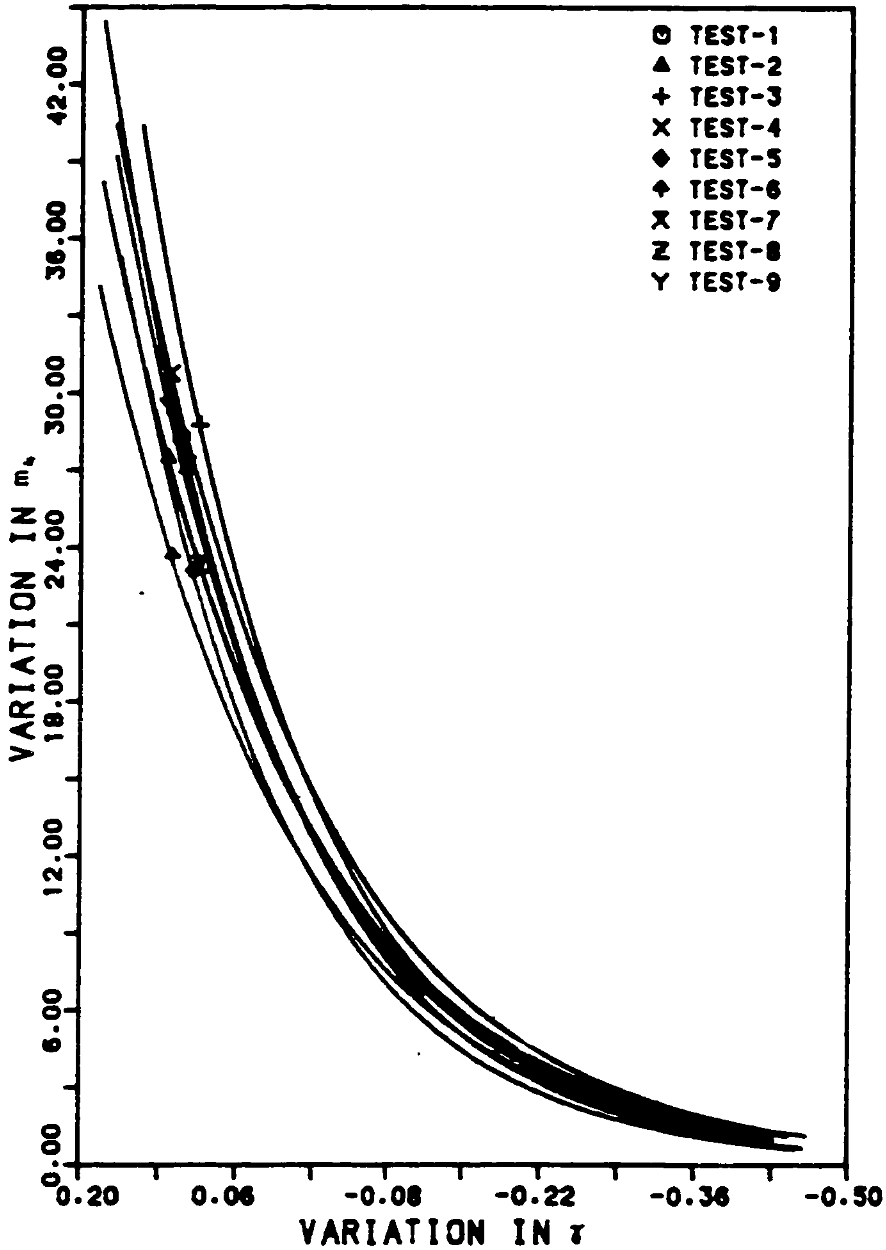


Figure 5.16. Variation in m_4 with Respect to Parameter γ

5.5 EFFECTS OF MACHINING CONDITIONS ON SURFACE ROUGHNESS:

The surface roughness models developed in the previous sections give the relationship between the individual effects of cutting speed V, feed rate F and depth of cut D, on the responses R_a, m_0, m_2 and m_4. For example, from the parameters of R_a model listed in Table 5.26, it is possible to observe the individual effects of the cutting conditions on R_a. The value of the power of V given by $\alpha = -9.587$, indicates that in the given range of the speed V, R_a will decrease, when the cutting speed is increased. On the other hand the power of feed-rate F, $\beta = 1.18$ and the power of depth of cut D, $\gamma = 0.16$ indicate that R_a increases with the increase of both feed-rate and depth of cut. However, β and γ have less effects on R_a than α. Similarly it is possible to arrive at the conclusions about the individual effects of the cutting conditions on m_0, m_2 and m_4 by using the average values of their model parameters. It is important to note that the individual effects of V, F and D may not be the only effects on the responses. There are other two-factors (FV, FD and VD) and the three-factor FVD effects, which result from the interaction of V, F and D. Therefore, the objective of this section is directed towards obtaining the main effects, two-factors effects and three-factors effects of V, F and D on R_a, m_0, m_2 and m_4.

The quantitative effects of cutting conditions (V, F, and D) and their interactions on R_a, m_0, m_2 and m_4 can be obtained from the 2^3 factorial design matrix experiments conducted in Chapter 4, which resulted in 150 responses at each of the 8 test conditions. The averages of the responses R_a, m_0, m_2 and m_4 computed from 150 profiles measured at each test were calculated at each test condition and listed in Table 5.37 versus their respective cutting conditions. The main effects of V, F, and D and the interaction of VF, VD, FD and VFD on R_a, m_0, m_2 and m_4 estimated by Yate's algorithm [45] from the data of Table 5.37 are listed in Table 5.38. The standard error of the effects corresponding to each response was also calculated and listed in Table 5.38.

The influence of the main effects (V, F and D) and their interactions on R_a is investigated first. Over the ranges of the variables studied, the main effects V, F and D and the interactions FV, FD and VD were found to be the effects distinguishable from noise. Table 5.37 reveals that when feed is increased from 0.2 mm/rev to 0.4 mm/rev, the increase in R_a is 72.81 percent to 91.32 percent. The Table also indicates that increase in speed from 100m/min to 215 m/min resulted in a decrease of R_a between 8.78% to 63.69%. Similarly, an increase of D from 0.3 mm to 0.77 mm resulted in an increase of R_a between 0.5 percent to 21.78

TABLE 5.37 Average Responses at Each Test Conditions

Test No.	Cutting conditions F	V	D	Average Response of R_a (μm)	m_0 $(\mu m)^2$	m_2	m_4 $(1/\mu m)^2$
1	-	-	-	2.19	0.79	41.06	23113.77
2	+	-	-	9.76	0.77	125.59	17071.56
3	-	+	-	.81	0.87	40.29	33356.21
4	+	+	-	9.44	0.85	123.28	20242.25
5	-	-	+	2.80	0.78	40.49	26954.55
6	+	-	+	10.30	0.76	123.86	10947.03
7	-	+	+	.98	0.86	39.74	23965.95
8	+	+	+	9.40	0.84	121.56	16594.33

Cutting Conditons	Code -	+
Feed,F (mm/rev.)	0.2	0.4
Speed,v (m/min.)	100.0	215.0
Depth of cut,D (mm)	0.3	0.7

Table 5.38 Effects of the Cutting Conditions and Their Interactions on R_a, m_0, m_2 and m_4

Test Identity No.	Effect	Effects on R_a (µm)	$m_0 * 10^3$	m_2 $(\mu m)^2$	m_4 $(1/\mu m)^2$
1	Average	5.715 ± .01	815.910 ± 0.15	81.986 ± 0.3	21530.69 ± 1248.0
2	F	8.031 ± .03	-19.908 ± 0.30	83.178 ± 0.6	-10633.83 ± 2496.0
3	V	-1.101 ± .03	77.998 ± 0.30	-1.531 ± 0.6	4017.95 ± 2496.0
4	FV	0.497 ± .03	-0.948 ± 0.30	-0.775 ± 0.6	391.04 ± 2496.0*
5	D	0.318 ± .03	4.994 ± 0.30	-1.144 ± 0.6	-3830.48 ± 2496.0
6	FD	-0.068 ± .03	-0.049 ± 0.30*	-0.580 ± 0.6*	-1055.74 ± 2496.0*
7	VD	-0.255 ± .03	-0.246 ± 0.30*	0.009 ± 0.6*	-2688.60 ± 2496.0
8	FVD	-0.034 ± .03*	-0.006 ± 0.30*	0.004 ± 0.6*	3926.91 ± 2496.0

* Confidence interval of the effects include zero and therefore they are attributed to the chance causes only and consequently they are negelected.

percent. However, since these variables interact with each other (FV = 0.49 $\pm$ 0.03, FD = -0.07 $\pm$ 0.03 and VD = 0.25 $\pm$ 0.03) V, F and D must be analysed jointly. In other words, it is not possible to predict the change in R_a by changing one of the cutting conditions separately.

The influence of the main effect and their interactions on m_0 is, however, different from the influence on R_a. The effects of F, V, D and FV are distinguishable from noise. As F interacts with V (FV interaction is -0.0009 $\pm$ 0.0003) their effects have to be interpreted jointly. However, the effect of D (+ 0.005 $\pm$ 0.0003) can be analysed separately. The influence of the main effects and their interactions on m_2 is similar to the influence on m_0, whereas for m_4 response, the comparison of their estimates with standard error suggests that the effects of F, V, D, and FVD are distinguishable from noise and require interpretation. Since the 3 factor interaction VFD = 3926.9 $\pm$ 2496.65) have appreciable effect, the effects V, F and D are analysed jointly. It is not possible to predict the change in m_0 by changing one of them separately.

The above discussion shows that most of the two-factor and three-factor interactions have a significant influence on the surface roughness responses R_a, m_0, m_2 and m_4. This indicates that the influence of the individual cutting variable cannot be considered separately when these interactions exist.

For example, in m_4 model, we cannot make any statement about the effects of V, F and D individually, because the three-factor interaction FVD has a significant effect on m_4.

It is important at this point to backtrack and emphasize the fact that the objective of this section was to evaluate the main effects and the two and three-way interactions of the cutting conditions on R_a, m_0, m_2 and m_4. Since the preceding analysis has indicated that the two-way and three-way interactions are significant, it may be interesting to fit higher order models which take care of this higher order effects and compare the results with the postulated model of Equation 5.1.

6. CONCLUSIONS AND RECOMMENDATIONS

6.1 SUMMARY AND CONCLUSIONS

1. A large number of models which are used in metal cutting industry are reviewed. This literature review has demonstrated the need for developing a technique that can fit models whose parameters are random variables.

2. A statistical modelling technique which treats the model parameters as random variables have been developed. The technique uses experimental data to compute the moments of the model parameters from the process response measured at a number of process independent variables. A methodology has been developed to identify the probability distributions of each model parameters from their corresponding computed moments.

3. To facilitate the implementation of the statistical modeling technique a special computer program package has been developed. The package is capable of identifying the distributions of the parameters of a linear model of the form:

$$Y = a_0 + a_1X_1 + a_2X_2 + a_3X_3$$

or of a simple exponential model of the form:

$$R = Kz_1^{a_1}\ z_2^{a_2}\ z_3^{a_3}$$

where Y and R are the depended random variables, X_1, X_2, X_3, z_1, z_2, z_3 are the independent process variables and a_0, a_1, a_2, a_3 are the statistically independent random parameters of the models.

4. The statistical modeling technique has been used to identify the probability distributions and estimate the corresponding distribution parameters from the simulated data of two different types of models. One of the models has been selected to have the same probability distribution for each of its four random parameters whereas the second model has been chosen to have different types of probability distributions for its parameters. The new technique was capable of both identifying the assumed distributions and obtaining approximately the same values of their respective distribution parameters.

5. Experimental work which resulted in fine turning surface roughness data was performed. A study was made to obtain the optimum sampling interval to digitize the surface profile data. The smallest optimum sampling interval

was found to be 3 μm in order that the digitized profiles could contain complete information of the surface roughness features.

6. Four surface roughness models were developed for the turning operation by using the new statistical modeling technique and the experimental surface roughness data. The developed models which express the relationship between the surface profile characteristics R_a, m_0, m_2 and m_4 and the operating conditions speed, feed and depth of cut are of the following form

$$R = K\, V^{\alpha} F^{\beta} D^{\gamma} \qquad \text{(I)}$$

and

$$Y = C + \alpha \ln V + \beta \ln F + \gamma \ln D \qquad \text{(II)}$$

The probability distributions of the parameters of R_a, m_0 and m_2 models defined by Equation (I) were found to be all normal whereas the probability distributions of C, α, β and γ of the m_2 model (Equation I) were found to be lognormal, normal, beta and normal respectively. A method has been developed to obtain the distribution of the random variable K (Equation II) from the distribution of C (Equation I). The distributions of K were found to be lognormal for R_a, m_0, and m_4 models.

7. The distributions of surface profile characteristics parameters R_a, m_0, m_2 and m_4 and their logrithms are identified. The distribution of R_a was found to be lognormal while the literature reports the divided results of normal, Weibull, etc. The distributions of m_0, m_2 and m_4 were found to be lognormal, gamma and lognormal respectively and have not been reported in the literature.

8. The new models of R_a, m_0, m_2 and m_4 were compared with the corresponding models developed using the conventional least square technique. The results showed that the values of the model parameters estimated by the conventional technique are approximately equal to the first moments about the origin of the respective parameters computed from the statistical modeling technique. This shows that the conventional technique is limited to estimate only the mean value of the parameters, whereas the proposed statistical modeling technique offers the method to compute the higher order moments of each model parameters and then to identify the distributions of the corresponding model parameters.

9. A study was made to obtain both the main effects and the two-way and three-way interactions of V, F and D on the response, R_a, m_0, m_2 and m_4.

The obtained results indicate that for R_a, m_0, m_2 and m_4, the two-way and three-way interactions are significant as compared to the individual effects on the responses.

6.2 RECOMMENDATIONS FOR FUTURE WORK

1. Since the developed computer package handle three process variables model only, a generalized computer program which can handle large number of process variable is required.
2. So far, only seven distributions have been included for identifying adequate probability distributions of the model parameters. It is recommended to include other distributions such as Weibull etc., in the moment ratios method.
3. Since on applying the moment ratios criterion eight moments have arbitrarily been chosen for identifying a parameter distribution, the optimum number of moments required for identification have to be studied to avoid excessive computations.
4. The proposed statistical modeling technique can be used for developing the other machining process models, such as, cutting force, tool life, torque, and power etc.

5. Higher order models are suggested for surface roughness to observe the interaction effects on the surface profile characteristic parameters.

6. The developed moment ratios criterion is proposed for use in the identification of probability distributions of any random variables in such applications as modeling and reliability analysis of life data, etc.

7. Since the statistical modeling technique has been developed for linear models, the extension of the technique to handle nonlinear models is proposed.

REFERENCES

1. Taylor, F.W., ' On the Art of Cutting Metals', Trans. ASME, Vol.28, (1907), PP.31.

2. Wu, S.M., 'Tool Life Testing by Response Surface Methodology Part I and II', Trans. ASME, Vol.86, (1964), PP.105.

3. Tipnis, V.A., and Friedman, M.Y., 'Cutting Rate - Tool Life Characteristic Functions for Material Removal Predict Surface Finish in Fine Turning of Steel', Int. J. Prod. Res., Vol.19, No.5, (1981), PP.556-574.

4. Levi, R., and Rossetto, S.,'Machining Economices and Tool Life Variation', Trans. ASME, Vol.100, (1978),PP.393-396.

5. Hasegawa, M., Seireg A., and Lindberg,. R.A, 'Surface Roughness Model for Turning', Tribology International, December, (1976), PP.286-289.

6. Nassirpur, F., and Wu, S.M., ' Statistical Evaluation of Surface Finish and its Relationship to Cutting Parameters in Turning', Int. J. Mach. Tool Des. and Res., Vol.17, (1977), PP.197-208.

7. Sundram, R.M. and Lambert, K., 'Mathematical Models to Predict Surface Finish in Fine Turning of Steel', Int. J. Prod. Res., Vol.19, No.5, (1981), PP.556-574.

8. Taraman, K., 'Surface Finish Prediction in Computerized machinability and System', ASME Tech. Paper MR78-654, (1978), PP.1-18.

9. Terman, K., and Lambert, B., 'A Surface Roughness Model for Turning Operation', Int. J. Prod. Res., Vol. 12, No.6, (1974), PP.691-703.

10. Bhattacharyya, A., and Faria, R., Gonzalez and Inyong Hong, 'Regression Analysis for Predicting Surface Finish and its Application in the Determination of Optimum Machining conditions', J. Engg. Ind. Trans. ASME, Vol.92, (1970), PP.711-714.

11. Taraman, K., Multi Machining - Multi Independent Variable Turning Research by Response Surface Methodology', Paper Presented at the 2nd International Conference on Production Research (Copenhagen) August, (9173).

12. Basuray, P.K., Sahay, B., and Lal, G.k., 'A Simple Model for Evaluating Surface Roughness in Fine Grinding', Int. J. Mach Tool Des. and Res., Vol.20, (1980), PP.265-273.

13. Sofronas, A., and Tarman, S., 'Model Development and Optimization of Vibratory finishing Process', Int. J. Prod. Res., Vol.17, No.1, (1979), PP.23-31.

14. Moltrent, K.H., 'Estimating Cutting Force', SME Tech Paper MRS885 (1976), PP.186-192.

15. Nakayama, K., and Arai, M., 'On the Storage of Data on Metal Cutting Forces', Annals of the CIRP, Vol. 25, Part 1, (1976), PP186-192.

16. Lambert, B.K., Dudek, R.A., and Williams, S.L., 'Single Point Tool Orthogonal Cutting Force Equations as a Function of Cutting Speed, Feed, Depth of Cut and Single Rake Angle Int. J. Prod. Res. Vol.6, No.3, (1968), PP.76-84.

17. Lambert, B.K., and Taraman, K., 'Development and Utilization of a Mathematical Model for a Turning Operation', Int. J. Prod. Res., Vol.6, No.3, (1971), PP.63-69.

18. Milner, D.A., and Raffat, H., 'An Experimental Investigation of the Variation of the Peak Torque and Power in Gear Hobbing Int. J. Prod. Res., Vol.14, No.5, (1976), PP.569-581.

19. Lamber, B.K., 'Prediction of Force, Torque and Burr Length in Drilling', SME Tech. Paper MR79-363, (1979), PP.1-10.

20. Devries, Marvin F., 'Carbide Cutting Tool Wear Models', SME Tech. Paper MR73-912, (1973), PP.1-8

21. Lipson, C., and Sheth, N.J., Statistical Design and Analysis of Engineering Experiments, McGraw Hill Book Company, (1973).

22. Fraser, D.A.S., 'Data Transformation and the Linear Model', Annals of Math. Statist., Vol.38, (1967), PP.1456-1465.

23. Doyle, Lawrece E., 'Optimum Cutting Speed and Tool Replacement Policy with Variable Tool Life', SME Tech. Paper MRS73-913, (1971), PP.1-18.

24. Sata, T., 'Surface Finish in Metal Cutting', Annals of the CIRP, Vol.12, Part 1, (1964), PP.190-199.

25. Perkelharing, J., and Gieszen, G.A., 'Material Side Flow in Finish Turning', Annals of the CIRP, Vol.20, Part 1, (1971), PP.21-33.

26. Selvon, M.S., and Balkrishnan,k., 'The Study of Machined Surface Roughness by Random Analysis' Wear, Vol.41, (1977), PP.287-293.

27. Tipnis, Vijay A., 'Mathematical and Economic Models for Material Removal Processes', Proc. of the 25th Sagamore Army Material Rearch Conference, held in July 17-21, 1978, Sagamore Hotel, Lake George, New York.

28. Beck, James V., and Arnold, Kenneth J., Parameter Estimation in Engineering and Science, John Wiley and Sons, (1977).

29. Khazonic, Rambart, Basic Probability Theory and Applications, Goodyear Publishing Company, Inc. (1976)

30. Patel, Jagdish K., Kapadia, H., and Own, D.B., Handbook of Statistical Distributions, Marcel Dekker, Inc. (1976)

31. Johson, Normal L., and Kotz, Samuel, Continuous Univariate Distributions -I, John Wiley and Sons, (1970)

32. Pandit, S.M., Nassirpour, F., and Wu, S.M., 'Stochastic Geometry of Anisotropic Surface with Application to Coated Abbrasive', J. Engg. Ind. Trans. ASME, Vol.99, (1977), PP.218-224.

33. Nassirpour, F., 'Characterization of Stochastic Surface Geometry with Applications', Ph.D. thesis, University of Wisconsin, Madison (1976).

34. Tsukada, Tadao and Sasajima, Kozuyuki, 'A Three Dimensional Measuring Technique for Surface Asperities', Wear, Vol.71, (1981), PP.103-111.

35. Shuaib, A.R.N., 'Computer Evaluation of Surface Topography (CEST) and Applications to Steel Finishing Processes', Ph.D. thesis, University of Wisconsin, Madison (1980)

36. Whitehouse, D.J., et al., 'Assesment of Surface Topography Analysis Techniques in Turning', Annals of the CIRP, Vol.23, Part 2, (1974), PP.641-648.

37. Newland, D.E., 'Random Vibration and Spectral Analysis', Longman Group Limited (1975).

38. Bendat, Julius S., and Pievsol, Allan G., 'Applications of Correlation and Spectral Analysis',John Wiley and Sons (1980)

39. Peters, J., Vanherek, P., Sastrodinoto, m., 'Evaluation of Surface Describing Parameters', Annals of the CIRP, Vol.20, Part 1, (1980), PP.352-357.

40. Law, Averill M., and Kelton W. David, Simulation Modeling and Analysis , McGraw-Hill Book Company, (1982).

41. Olsen, K.V., 'Surface Rougness as a Function of the Cutting Data when Fine Turning Steel', ASTM Technical Paper No. MR64-182, (1964), PP.1-16.

42. Allen, A.F., and Brew, R.C., 'The Influence of Machine Tool Vibration and Tool Flank Wear on Surface Texture', Adv. Mach. Tool Des. Vol.1, (1967), PP.310-307.

43. Petropolos, P.G., 'Statistical Basis for Surface Roughness Assessment in Oblique Finish Turning steel Components', Int. J. Prod. Res., Vol.12, No.2, (1974), PP.345-351.

44. Sundram, R.M., and Lambart, B.K., 'Surface Roughness Variability of AISI4140 Steel in Fine Turning Using Carbide Tools', Int. J. Prod. Res. Vol.17, No.3,(1979), PP.440-453

45. Box, George E.P., Hunter, William , and Hunter J.Stuart, Statistcs for Experiments , John Wiley & Sons (1978).

46. Erik Oberg, Franklin D. Jones and Holbrook L. Hoton, Machinery's Handbook, Revised 21st Edition, Industrial Press Inc., (1979).

APPENDIX A

MATHEMATICAL DERIVATION

Rewriting the Equation 3.18

$$M_Y(t) = M_{a_0}(t) \cdot M_{a_1}(X_1 t) \cdot M_{a_2}(X_2 t) \ \ldots \ M_{a_n}(X_n t) \quad (A.1)$$

Taking the first derivative w.r.t. t:

$$\begin{aligned} M'_Y(t) = & [M'_{a_0}(t) \ M_{a_1}(X_1 t) \ M_{a_2}(X_2 t) \ \ldots \ M_{a_n}(X_n t)] \\ & + [M_{a_0}(t) \ M'_{a_1}(X_1 t) X_1 \ M_{a_2}(X_2 2) \ \ldots \ M_{a_n}(X_n t)] \\ & + [\quad \ldots \quad \ldots \quad] \\ & + [\quad \ldots \quad \ldots \quad] \\ & + [M_{a_0}(t) \ M_{a_1}(X_1 t) \ M_{a_2}(X_2 t) \ \ldots \ M'_{a_n}(X_n t) X_n] \end{aligned}$$

(A.2)

Taking the second derivative

$$\begin{aligned} M''_Y(t) = & [M'_Y(t)]' \\ & + [\{M''_{a_0}(t) \ M_{a_1}(X_1 t) \ M_{a_2}(X_2 t) \ \ldots \ M_{a_n}(X_n t)\} \\ & + \{M'_{a_0}(t) \ M'_{a_1}(X_1 t) X_1 \ M_{a_2}(X_2 t) \ \ldots \ M_{a_n}(X_n t)\} \end{aligned}$$

$$+ \{M'_{a_0}(t)\ M'_{a_1}(X_1t)X_1\ M_{a_2}(X_2t)\ \ldots\ M_{a_n}(X_nt)\}$$

$$+ \{\qquad \ldots \qquad \ldots \qquad\}$$

$$+ \{\qquad \ldots \qquad \ldots \qquad\}$$

$$+ \{M'_{a_0}(t)M_{a_1}(X_1t)M_{a_2}(X_2)\ \ldots\ M'_{a_n}(X_nt)X\}]$$

$$+ [\{M'_{a_0}(t)M'_{a_1}(X_1t)X_1\ M_{a_2}(X_2t)\ \ldots\ M_{a_n}(X_nt)\}$$

$$+ \{M_{a_0}(t)M''_{a_1}(X_1t)X_1^2M_{a_2}(X_2t)\ \ldots\ M_{a_n}(X_nt)\}$$

$$+ \{M_{a_0}(t)M'_{a_1}(X_1t)X_1M'_{a_2}(X_2t)X_2\ \ldots\ M_{a_n}(X_nt)\}$$

$$+ \qquad \ldots \qquad \ldots$$

$$+ \qquad \ldots \qquad \ldots$$

$$+ \{M_{a_0}(t)M'_{a_1}(X_1t)X_1M_{a_2}(X_2t)\ \ldots\ M'_{a_n}(X_nt)X_n\}]$$

$$+ [\{\qquad \ldots \qquad \ldots \qquad\}$$

$$+ \{\qquad \ldots \qquad \ldots \qquad\}$$

$$+ \{\qquad \ldots \qquad \ldots \qquad\}]$$

$$+ [\{M'_{a_0}(t)M_{a_1}(X_1t)M_{a_2}(X_2t)\ \ldots\ M'_{a_n}(X_n)X_n)X_n\}$$

$$+ \{M_{a_0}(t)M'_{a_1}(X_1t)X_1M_{a_2}(X_2t)\ \ldots\ M'_{a_n}(X_nt)X_n\}$$

$$+ \{M_{a_0}(t)M_{a_1}(X_1 t)M'_{a_2}(X_2 t)X_2 \ \ldots \ M'_{a_n}(X_n t)X_n\}$$

$$+ \{ \qquad \ldots \qquad \ldots \qquad \}$$

$$+ \{ \qquad \ldots \qquad \ldots \qquad \}$$

$$+ \{M_{a_0}(t)M_{a_1}(X_1 t)M_{a_2}(X_2 t) \ \ldots \ M''_{a_n}(X_n t)X_n^2\}] \tag{A.3}$$

by setting $t = 0$

and

$$M_X(0) = 1$$

$$M'_X(0) = \mu'_{1X}$$

$$M''_X(0) = \mu'_{2X}$$

Equation A.3 becomes

$$\mu'_{2Y} = \{\mu'_{2a_0} + \mu'_{1a_0}\mu'_{1a_1}X_1 + \mu'_{1a_0}\mu'_{1a_2}X_2 + \ldots \mu'_{1a_0}\mu'_{1a_n}X_n\}$$

$$+ \{\mu'_{1a_0}\mu'_{1a_1}X_1 + \mu'_{2a_1}X_1^2 + \mu'_{1a_1}X_1\mu'_{1a_2}X_2 + \ldots \mu'_{1a_1}X_1\mu'_{1a_n}X_n\}$$

$$+ \{\mu'_{1a_0}\mu'_{1a_2}X_2 + \mu'_{1a_1}X_1\mu'_{1a_2}X_2 + \mu'_{2a_2}X_2^2 + \ldots \mu'_{1a_2}X_2\mu'_{1a_n}X_n\}$$

$$+ \{ \qquad \ldots \qquad \ldots \qquad \}$$

$$+ \{ \qquad \ldots \qquad \ldots \qquad \}$$

$$+ \{\mu'_{1a_0}\mu'_{1a_n}X_n + \mu'_{1a_1}X_1\mu'_{1a_n}X_n + \mu_{1a_2}X_2{}_{1a_n}X_n + \cdots \mu'_{2a_n}X_n^2\}$$

which can be simplified as

$$\mu'_{2Y} = \{\mu'_{2a_0} + \mu'_{2a_1}X_1^2 + \mu'_{2a_2}X_2^2 + \cdots \mu'_{2a_n}X_n^2\}$$

$$+ 2\mu'_{1a_0}\{\mu'_{1a_1}X_1 + \mu'_{1a_2}X_2 + \cdots \mu'_{1a_n}X_n\}$$

$$+ 2\mu'_{1a_1}X_1\{\mu'_{1a_2}X_2 + \mu'_{1a_3}X_3 + \cdots \mu'_{1a_n}X_n\}$$

$$+ \qquad \{ \qquad \qquad \qquad \}$$

$$+ \qquad \{ \qquad \qquad \qquad \}$$

$$+ 2\mu'_{1a_{n-1}}X_{n-1}\{\mu_{1a_n}X_n\} \tag{A.4}$$

which is the same as Equation 3.22.

APPENDIX B

PROGRAM FOR TRANSFERING THE DISCRETE PROFILE DATA FROM NICOLLET 2082 DIGITAL OSSCILLOSCOPE TO THE MEMORY OF TETRONIX 834 MICROCOMPUTER.

SETTING OF TEXTRONIX 834 PROGRAMABLE DATA COMMUNICATION TESTER.

SETUP

```
CODE     = ASCII
BAUD     = 9600
DUPLEX   = FULL
DELAY    =  200
SETUP    = ASYNC
BITS/CHR = 8
PARITY   = EVEN
STOPBITS = 1
EOF      = OA
TIMING   = NORMAL
```

TRIGER

```
1 SEND : 1
2 RECEIVE
3 HALT : 0
```

PROGRAM

```
MESSAGE : 3000
1M : < EMPTY >
```

01 $\overline{\text{ENTER}}$	45 $\overline{\text{ENTER}}$	32 $\overline{\text{ENTER}}$	44 $\overline{\text{ENTER}}$
44 $\overline{\text{ENTER}}$	30 $\overline{\text{ENTER}}$	4F $\overline{\text{ENTER}}$	32 $\overline{\text{ENTER}}$
30 $\overline{\text{ENTER}}$	30 $\overline{\text{ENTER}}$	02 $\overline{\text{ENTER}}$	

This program is equivalent to

```
SOH
E2
D1
DO
02000
STX
```

APPENDIX C

PROGRAM FOR TRANSFERING THE DISCRETE PROFILE DATA FROM TEXTRONIX 834 MICROCOMPUTER TO THE MEMORY OF HOST COMPUTER IBM-370/3033.

```
       INTEGER IBUF(200)
       INTEGER IRPS (24)/Z1B,Z4E,Z53,Z39,Z45,Z36,Z44,Z3D,
      *Z45,Z32,Z44,Z35,Z44,Z31,Z44,Z34,Z3D,Z3A,Z1B,Z32/
       CHARACTER*1 C(70)
       CHARACTER*7 D
C***  SET PROMPTSTRING IN TERMINAL
C***  (SEC)(N)(S)<INT-ARRY>
       CALL ADEOUT (24,IPRS)
       WRITE(20,111)(IPRS(I),I=1,9)
 111   FORMAT(20A4)
C***  SET PROMPTMODE ON TERMINAL
C***  (SEC)(N)(M)(2)
       CALL ERRSET(215,255,1,1,FRED,0)
       CALL INPUTN('ENTER C .',C)
       STOP
       END
C***
C***
       SUBROUTINE FRED(A,B,E)
       CHARACTER*1 E
       INTEGER*4 B
       INTEGER ERR215
       INTEGER ERR217
       COMMON/RRR215/ ER215
       LOGICAL*1 TABENT(8)
       IF(B.EQ.215)ER215=1
       E='O'
       RETURN
       END
C***
C***
       SUBROUTINE INPUTN(A,B)
       CHARACTER*1 A(75),B(75)
       CHARACTER*1 EE(2)
       EXTERNAL FRED
       INTEGER RRR215
       INTEGER ERR217
       COMMON/RRR215/ERR215
       EE(1)='/'
```

```
      CALL ERRSET(215,0,-1,1,FRED,0)
100   ERR215=0
      ERR217=0
      READ(5,11,END=999) B
      IF(B(1).EQ.EE(1).AND.B(2).EQ.EE(2)) GOTO 888
 11   FORMAT(70A1)
      WRITE(21,12) B
      CALL CLEAR
999   REWIND 5
      GOTO 100
 12   FORMAT(70A1)
888   STOP
998   RETURN
      END
```